CMB Non-Gaussianity: Modal Methods

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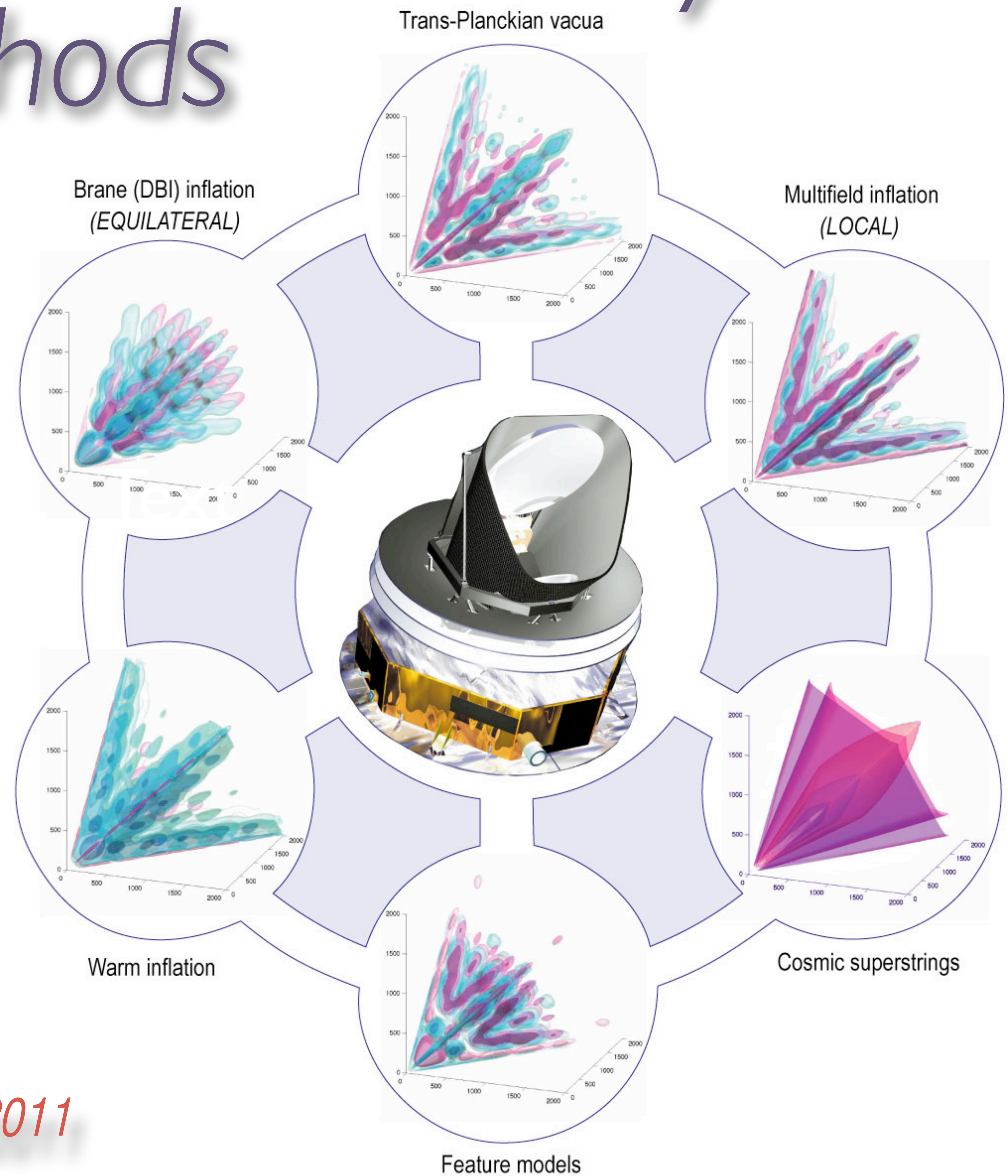
also Hiro Funakoshi and Marcel Schmittfull

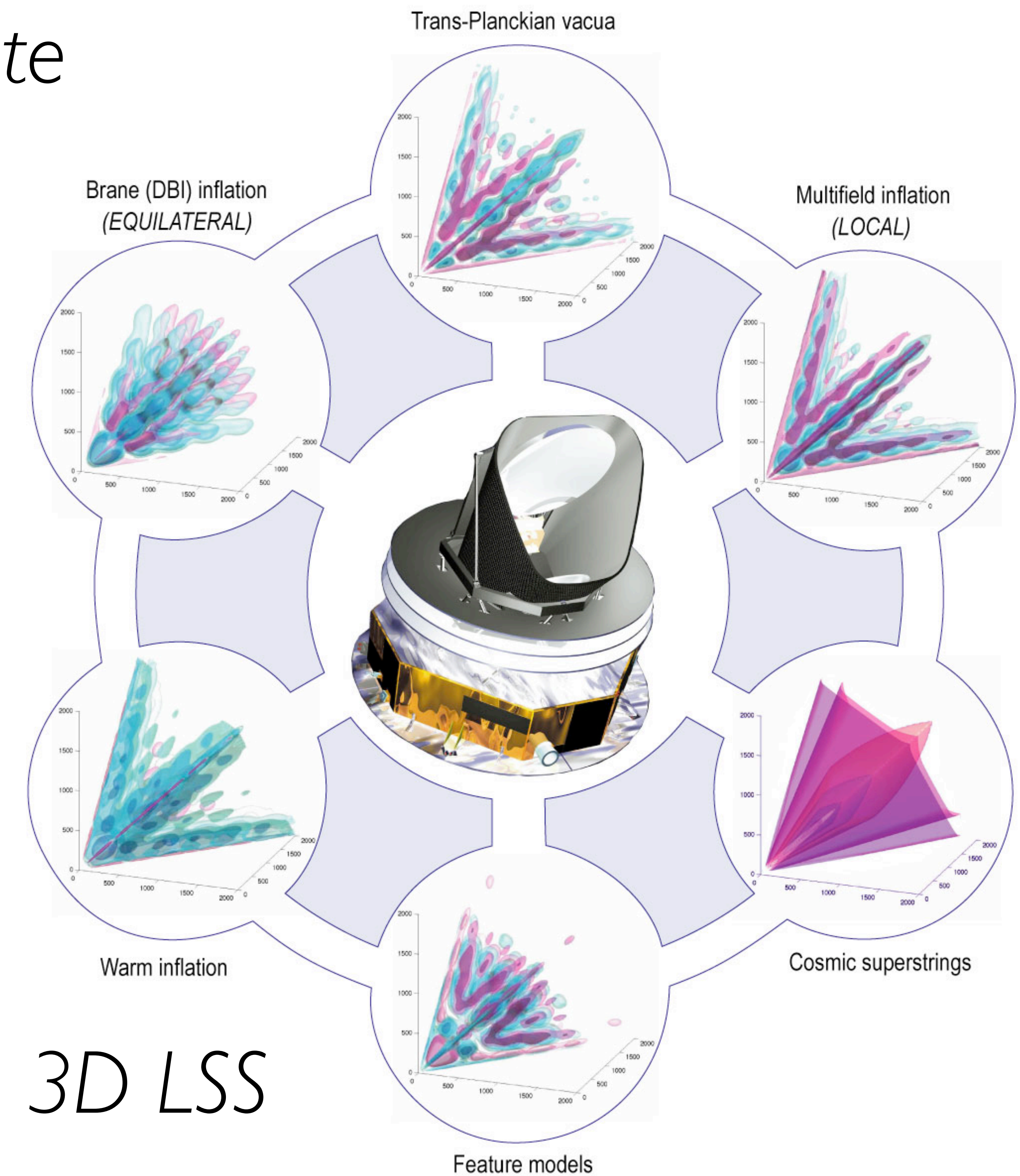
arXiv: today ...
arXiv:1006.1642
arXiv:1012.6039
arXiv:1008.1730
(arXiv:1004.2915), arXiv:0912.5516, arXiv:0812.3413, astro-ph/0612713

Michigan NG Workshop • May 2011



Modal motivations

- *Generic - early & late*
- *Highly efficient*
- *Reconstruction*
- *Blind survey*
- *Optimal*
- *Simulations*
- *Planck resolution*
- *CMB polyspectra & 3D LSS*



Background

- The primordial bispectrum and trispectrum* are defined by

$$\langle \Phi(\mathbf{k}_1)\Phi(\mathbf{k}_2)\Phi(\mathbf{k}_3)\rangle = (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)B(k_1,k_2,k_3)$$

$$\langle \Phi(\mathbf{k}_1)\Phi(\mathbf{k}_2)\Phi(\mathbf{k}_3\Phi(\mathbf{k}_4))\rangle = (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3+\mathbf{k}_4)T(k_1,k_2,k_3,k_4)$$

- For the CMB the bispectrum and trispectrum* are defined by

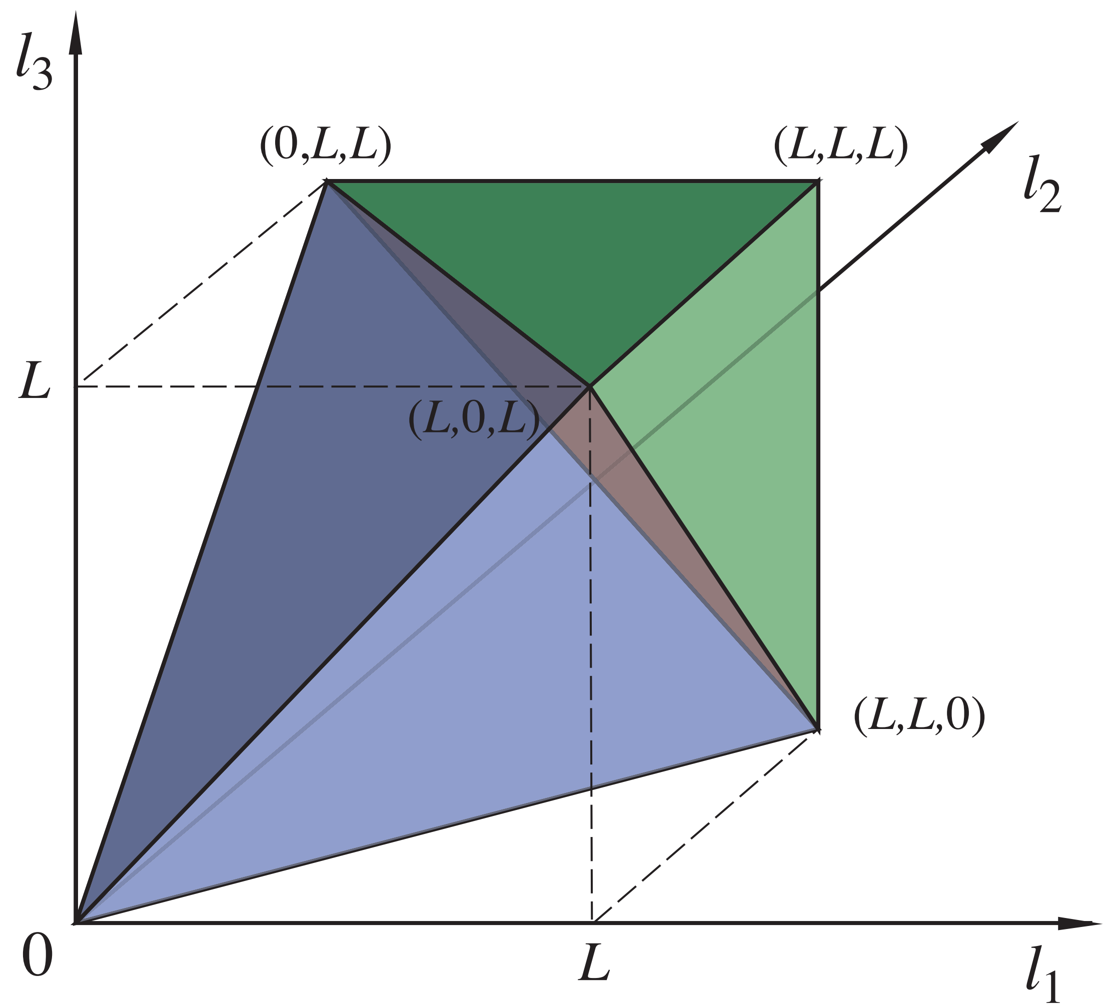
$$\langle a_{l_1m_1}a_{l_2m_2}a_{l_3m_3}\rangle = \left(\int d^2\hat{n} Y_{l_1m_1}(\hat{\mathbf{n}})Y_{l_2m_2}(\hat{\mathbf{n}})Y_{l_3m_3}(\hat{\mathbf{n}})\right) b_{l_1l_2l_3}$$

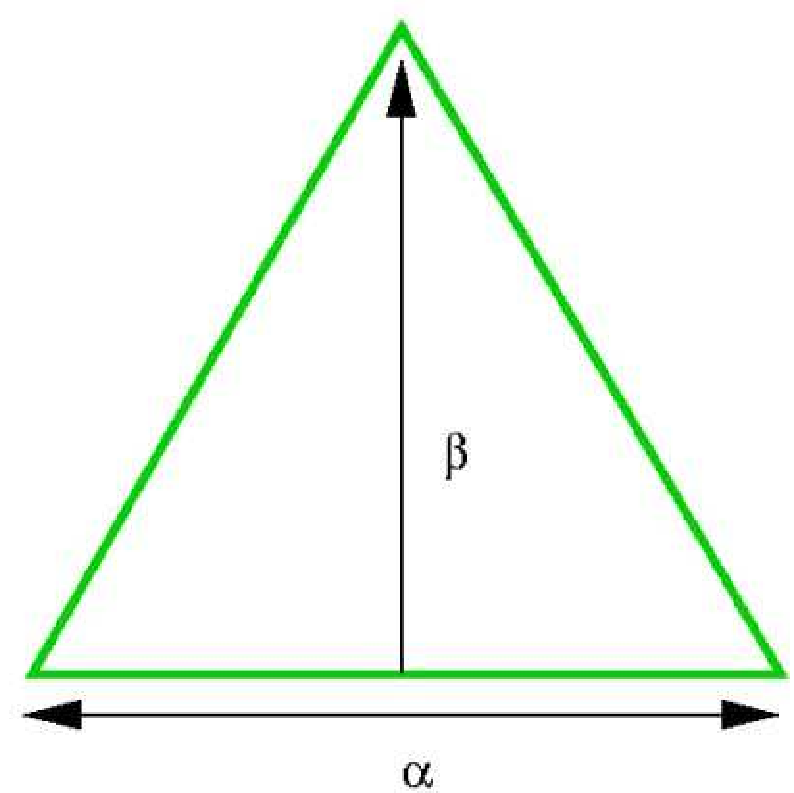
$$\langle a_{l_1m_1}a_{l_2m_2}a_{l_3m_3}a_{l_4m_4}\rangle = \left(\int d^2\hat{n} Y_{l_1m_1}(\hat{\mathbf{n}})Y_{l_2m_2}(\hat{\mathbf{n}})Y_{l_3m_3}(\hat{\mathbf{n}})Y_{l_4m_4}(\hat{\mathbf{n}})\right) t_{l_1l_2l_3l_4}$$

* For simplicity we give formulae only for diagonal-free trispectra.

TETRAPYD DOMAIN

There is a triangle condition on the three k or L so the domain for the bispectrum is a tetrahedron (tetrapyd)

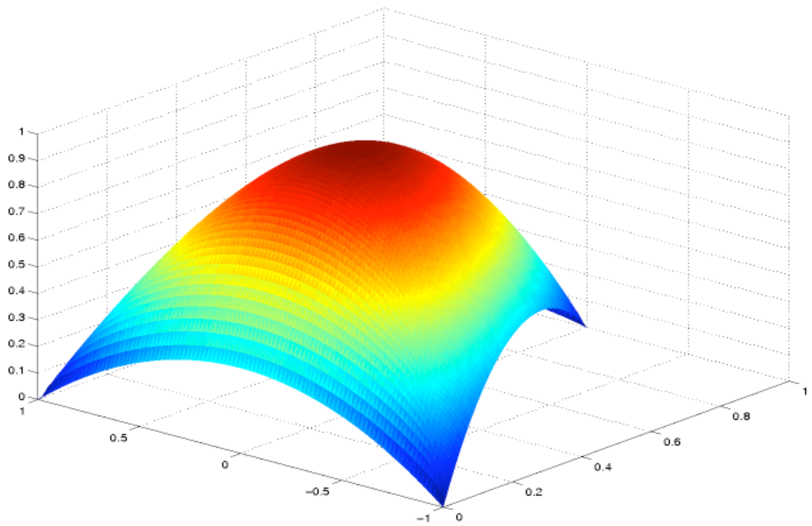


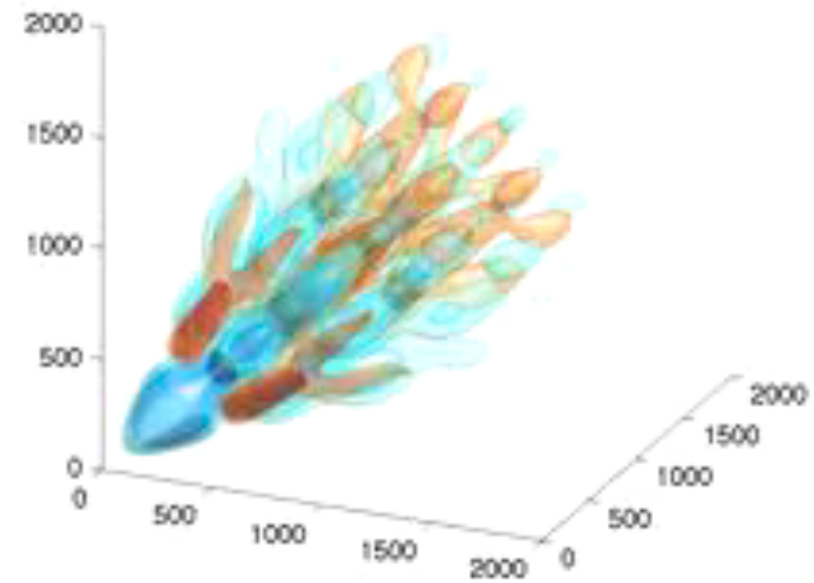


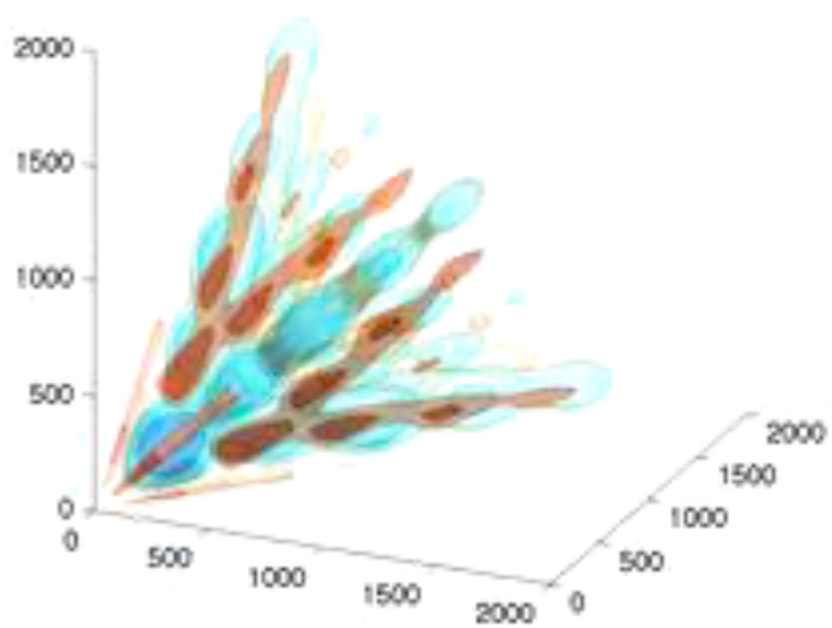
For scale-invariant bispectra (k^{-6}), we define the shape

$$S(k_1, k_2, k_3) \equiv \frac{1}{N}(k_1 k_2 k_3)^2 B_\Phi(k_1, k_2, k_3)$$

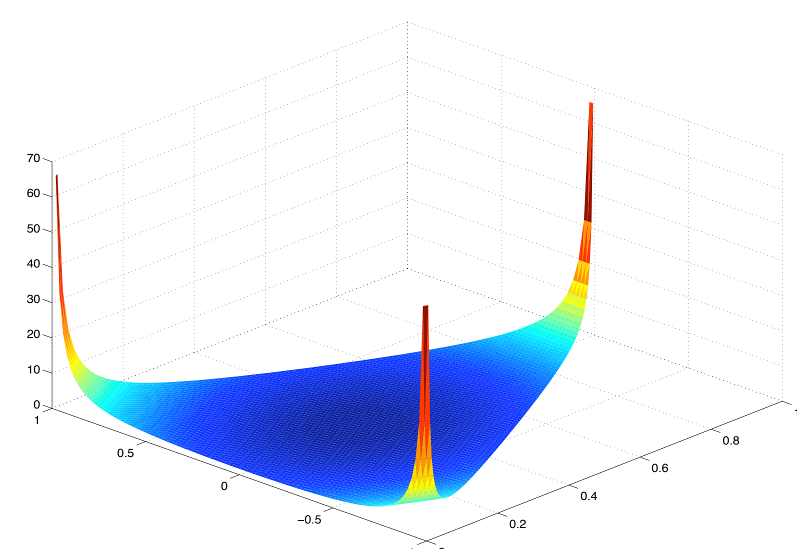
DBI Inflation »»»»»»







«««« Multifield Inflation



Many challenges

- **THEORETICAL** - To project the primordial correlators to their late time counterparts we must perform the integrals

$$b_{l_1 l_2 l_3} = \left(\frac{2}{\pi}\right)^3 \int x^2 dx \int dk_1 dk_2 dk_3 (k_1 k_2 k_3)^2 B(k_1, k_2, k_3) \Delta_{l_1}(k_1) \Delta_{l_2}(k_2) \Delta_{l_3}(k_3) j_{l_1}(xk_1) j_{l_2}(xk_2) j_{l_3}(xk_3)$$

$$t_{l_1 l_2 l_3 l_4} = \left(\frac{2}{\pi}\right)^4 \int x^2 dx \int dk_1 dk_2 dk_3 dk_4 (k_1 k_2 k_3 k_4)^2 T(k_1, k_2, k_3, k_4) \Delta_{l_1}(k_1) \Delta_{l_2}(k_2) \Delta_{l_3}(k_3) \Delta_{l_4}(k_4) j_{l_1}(xk_1) \ldots.$$

- **DATA ANALYSIS** - We endeavour to determine the goodness of fit between theory and the CMB with the estimator

$$\mathcal{E} = \sum_{l_i m_i} \frac{\langle a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3} \rangle^{f_{NL}=1} a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3}}{C_{l_1} C_{l_2} C_{l_3}}$$

$$= \sum_{l_i m_i} \frac{\int Y_{l_1 m_2} Y_{l_2 m_2} Y_{l_3 m_3} b^{f_{NL}=1}_{l_1 l_2 l_3} a_{l_1 m_1} a_{l_2 m_2} a^{l_i}_{l_3 m_3}}{C_{l_1} C_{l_2} C_{l_3}}$$

TRACTABILITY = SEPARABILITY

$$S(k_1, k_2, k_3) = X(k_1)\, Y(k_2)\, Z(k_3) + 5 \text{ perms.}$$

$$b_{\ell_1 \ell_2 \ell_3} = \int dr r^2 X_{\ell_1}(r)\, Y_{\ell_2}(r)\, Z_{\ell_3}(r) + \;5 \text{ perms}$$

$$\mathcal{E}(a) = \frac{1}{\mathcal{N}} \int dr\, r^2 \int d\Omega_{\hat{\mathbf{n}}} M_X(r, \hat{\mathbf{n}}) M_Y(r, \hat{\mathbf{n}}) M_Z(r, \hat{\mathbf{n}})$$

$$X_\ell(r) \equiv \int dk k^2\, X(k)\, j_\ell(kr)\, \Delta_\ell$$

$$Y_\ell(r) \equiv \int dk k^2\, Y(k)\, j_\ell(kr)\, \Delta_\ell$$

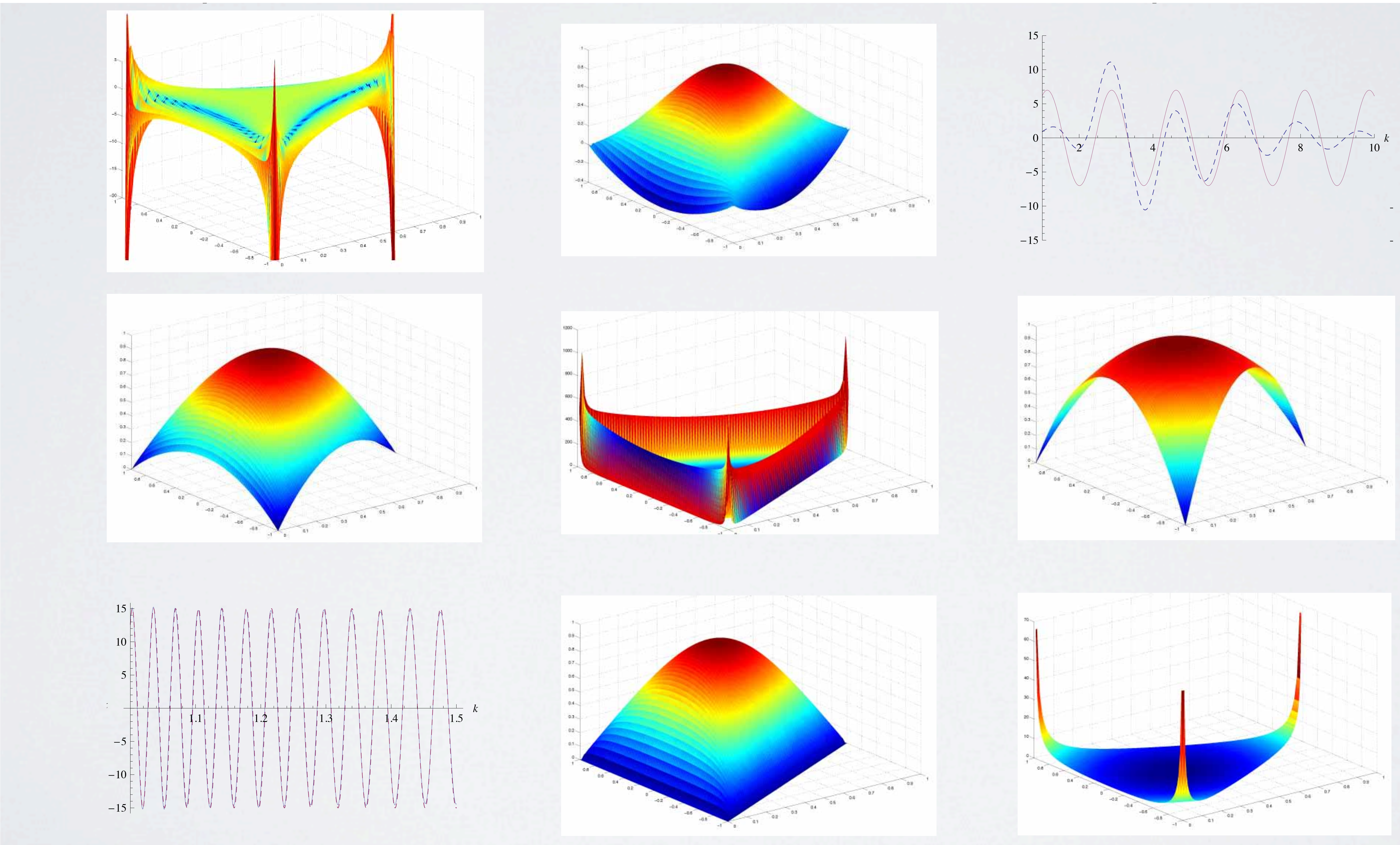
$$Z_\ell(r) \equiv \int dk k^2\, Z(k)\, j_\ell(kr)\, \Delta_\ell$$

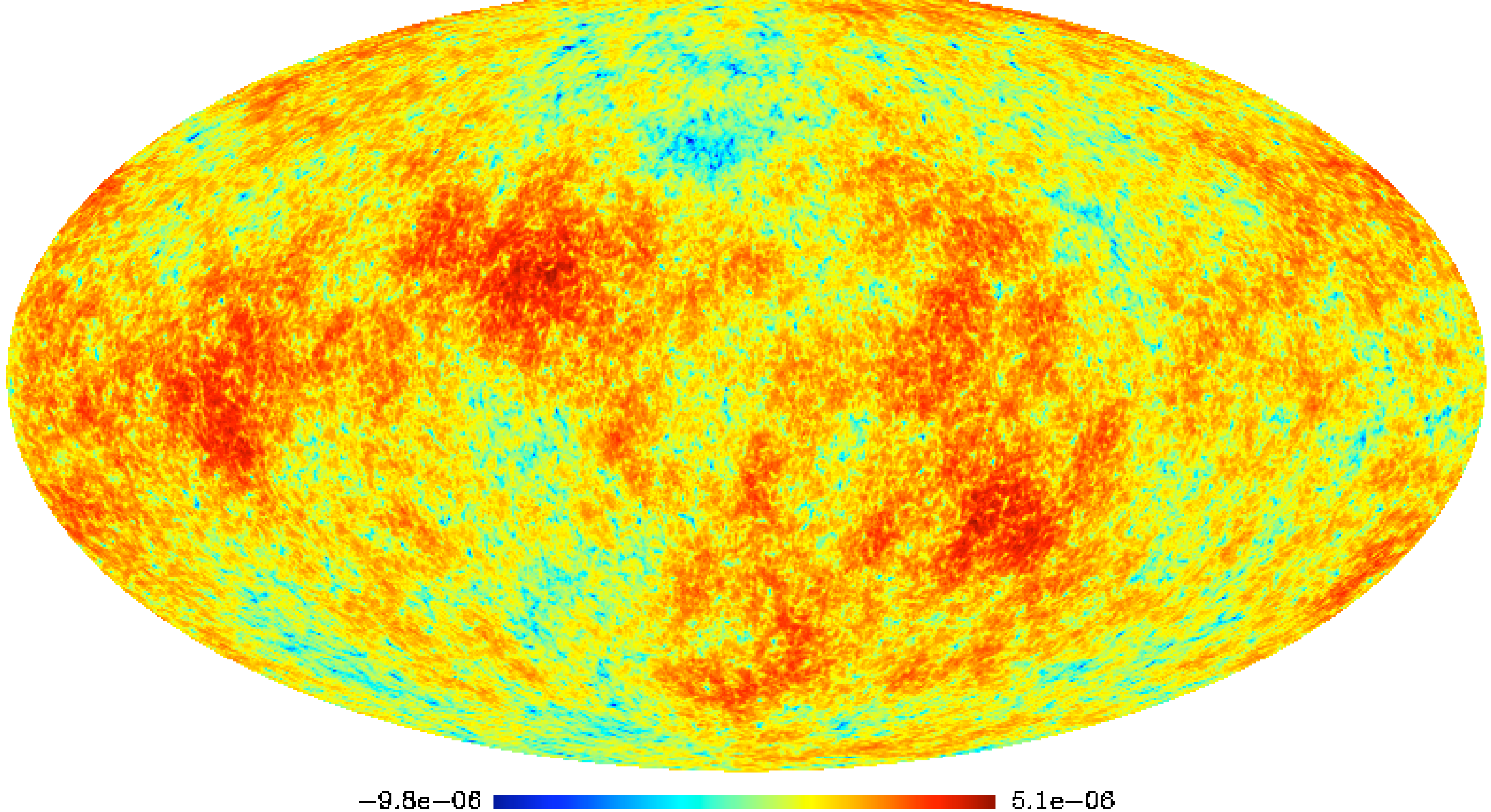
$$M_X(r, \hat{\mathbf{n}}) \equiv \sum_{\ell m} \frac{a_{\ell m} X_\ell(r)}{C_\ell} Y_{\ell m}(\hat{\mathbf{n}})$$

$$M_Y(r, \hat{\mathbf{n}}) \equiv \sum_{\ell m} \frac{a_{\ell m} Y_\ell(r)}{C_\ell} Y_{\ell m}(\hat{\mathbf{n}})$$

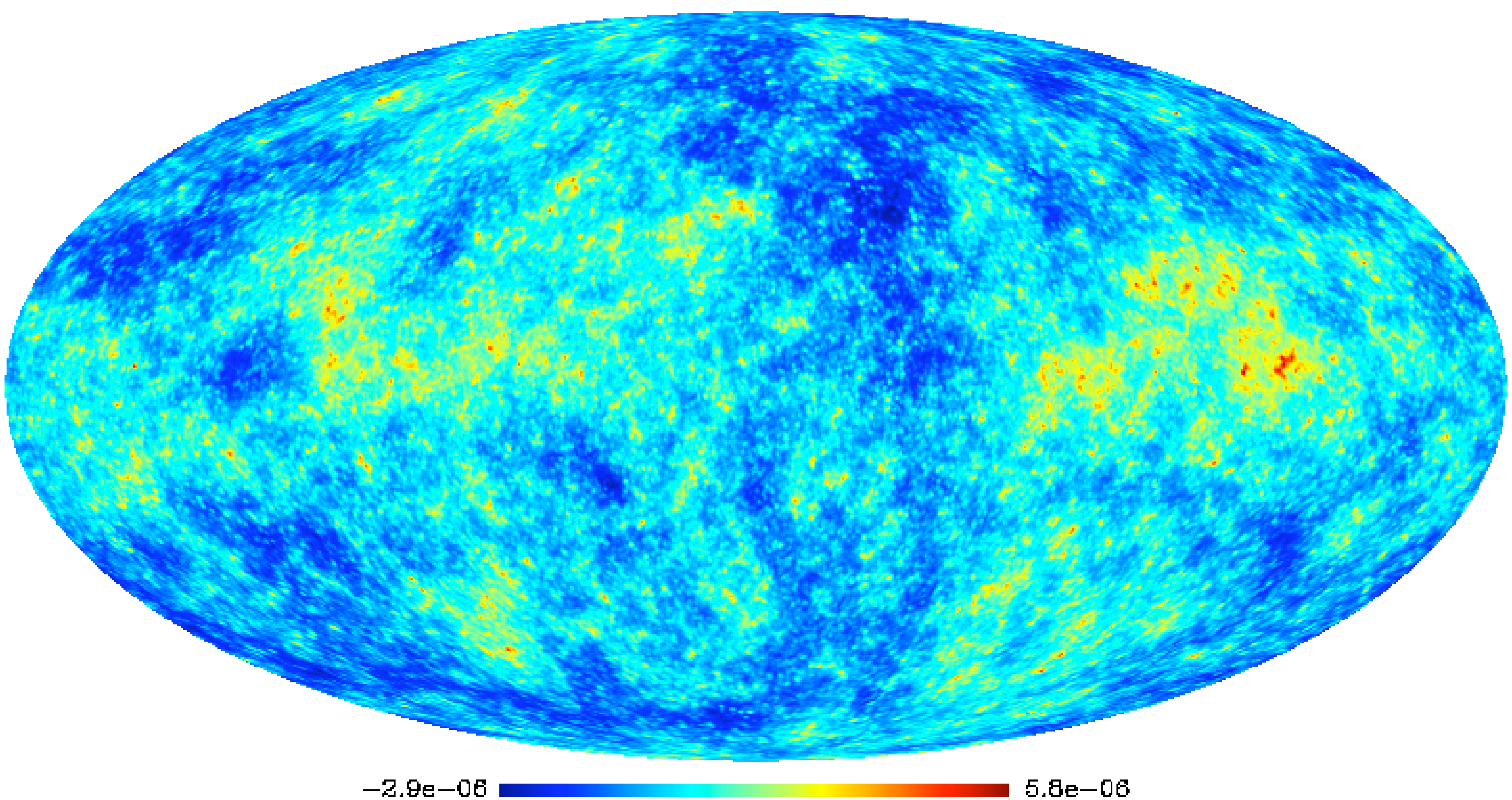
$$M_Z(r, \hat{\mathbf{n}}) \equiv \sum_{\ell m} \frac{a_{\ell m} Z_\ell(r)}{C_\ell} Y_{\ell m}(\hat{\mathbf{n}})$$

BUT there are many possible shapes predicted for the primordial bispectrum, *few* of which are separable





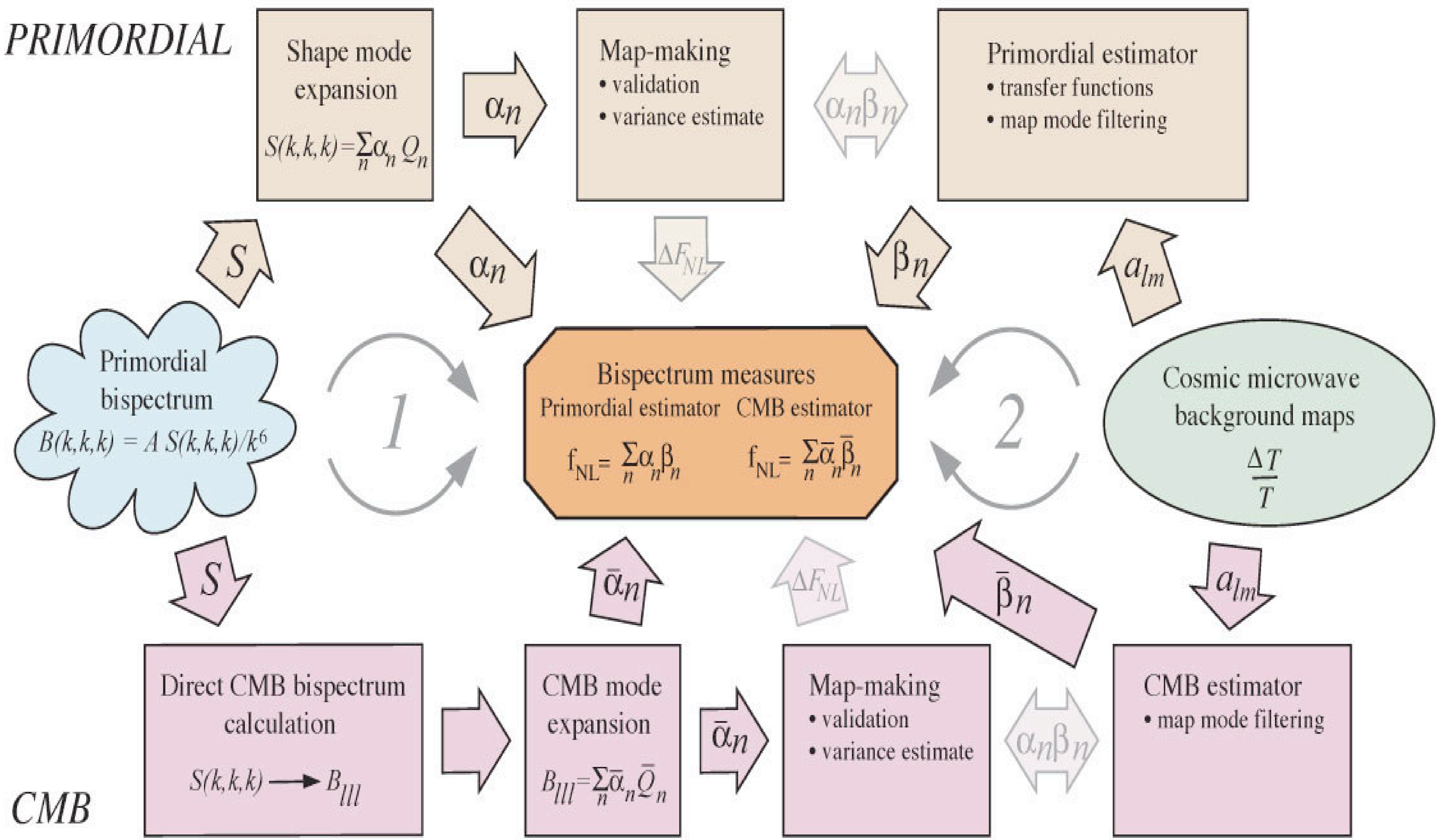
Cosmic strings



DBI inflation

General Bispectrum Estimator Pipeline

Fergusson, Ligouri and EPS arXiv: 0912.5516



MODAL POLYSPECTRA

PRIMORDIAL POLYSPECTRA

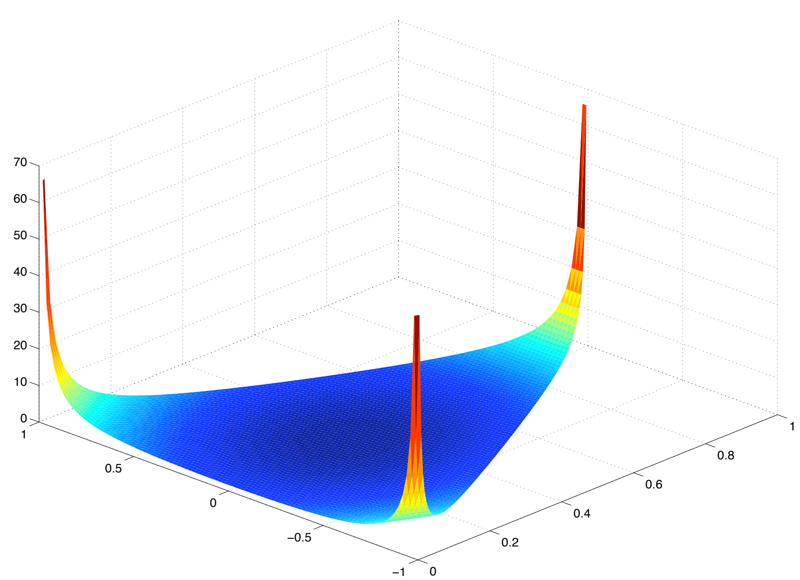
ESTIMATOR

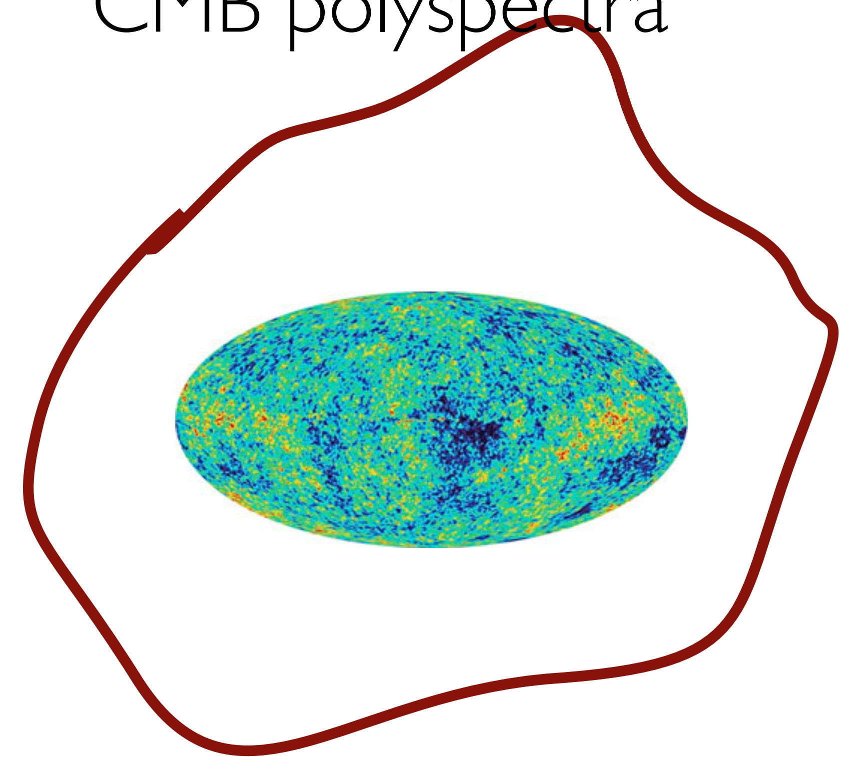
EVOLUTION BY TRANSFER FUNCTIONS

OBSERVED OR SIMULATED CMB MAPS

Primordial isotropic polyspectra space

Space *V of possible* CMB polyspectra





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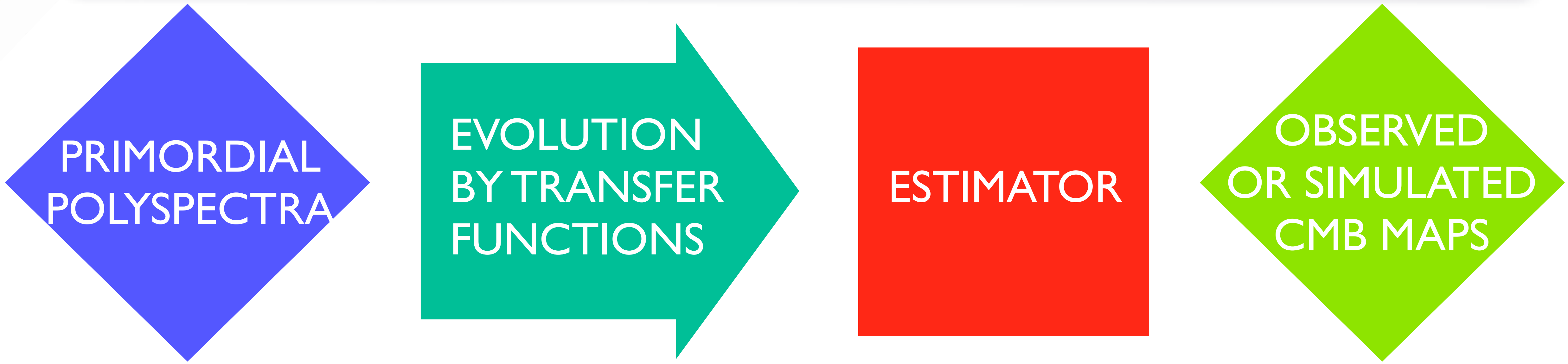

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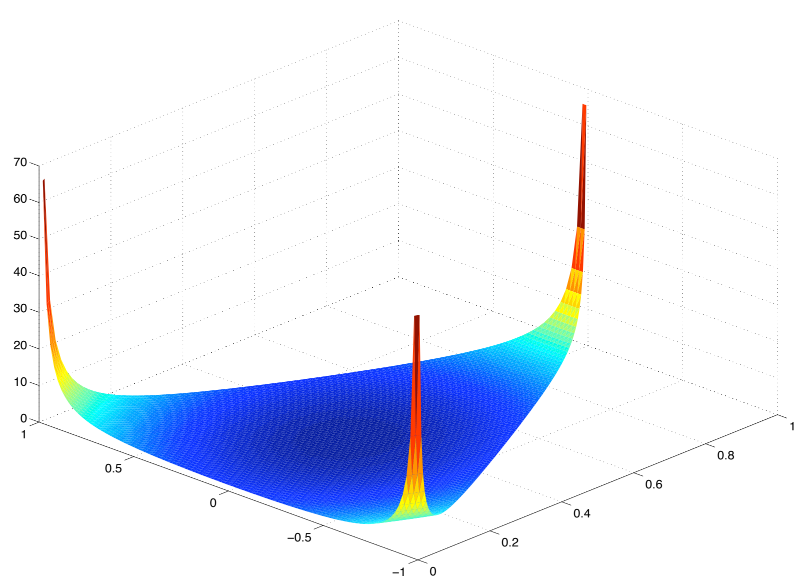

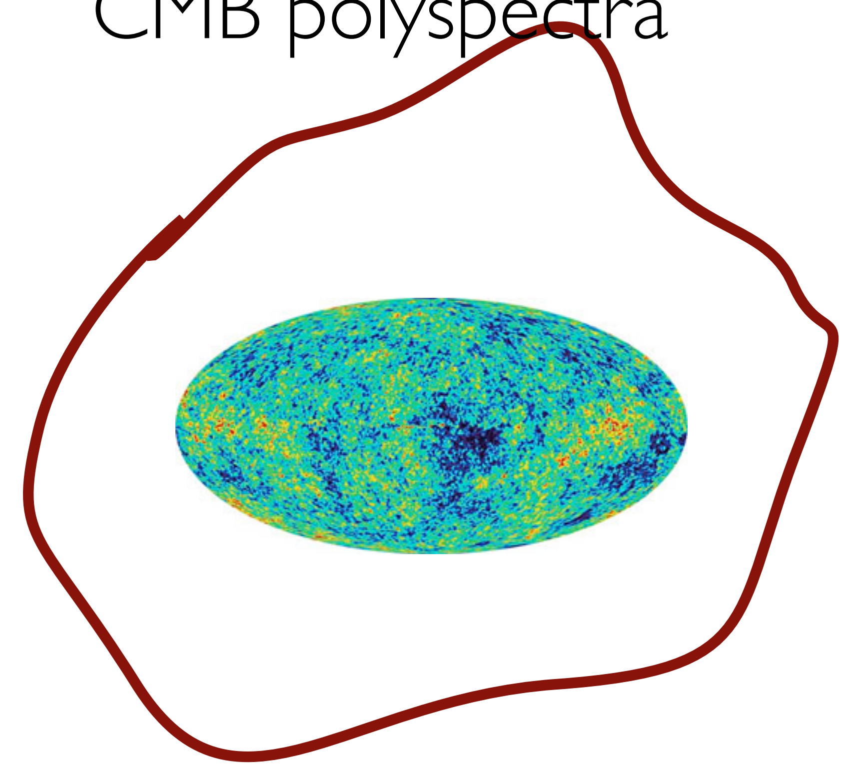

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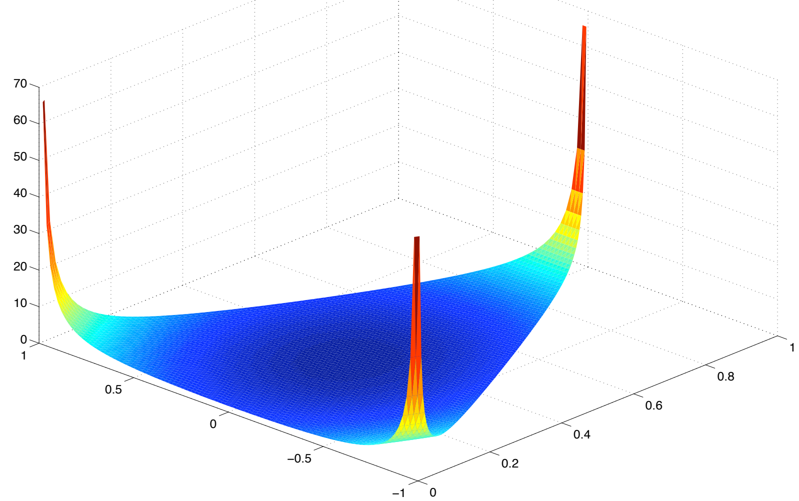
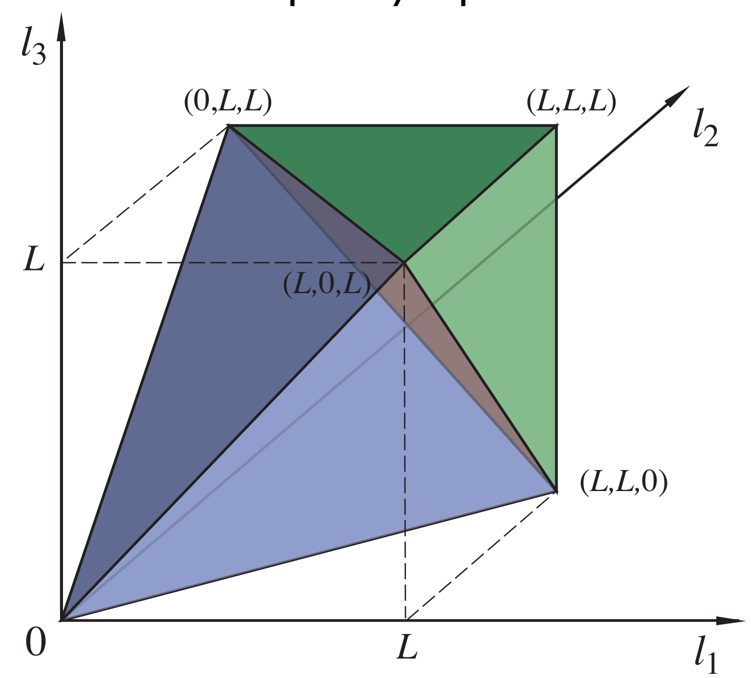


Projected space V_P of CMB polyspectra



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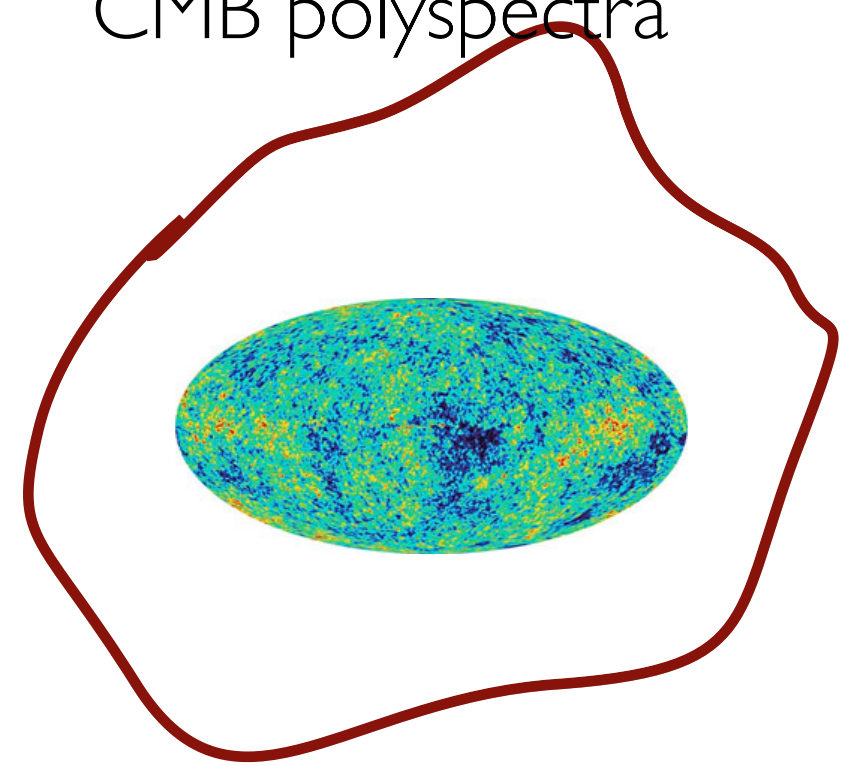

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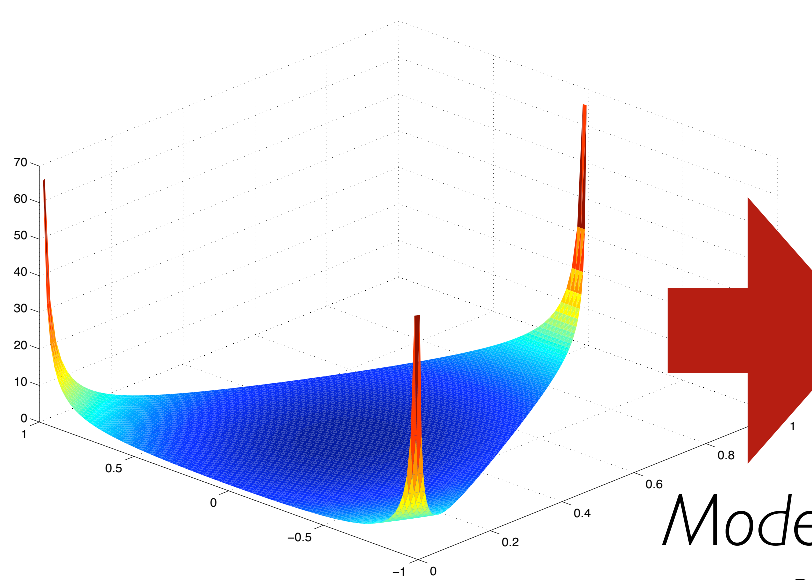
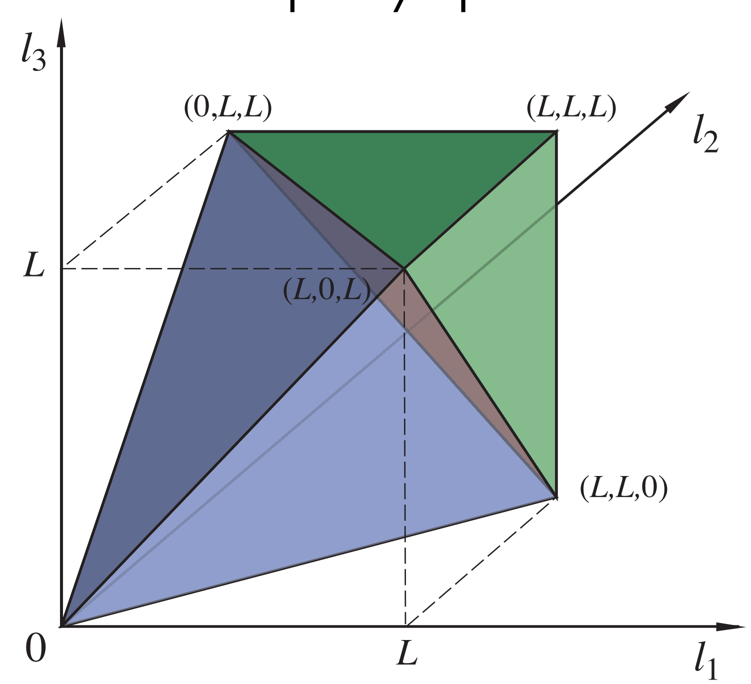
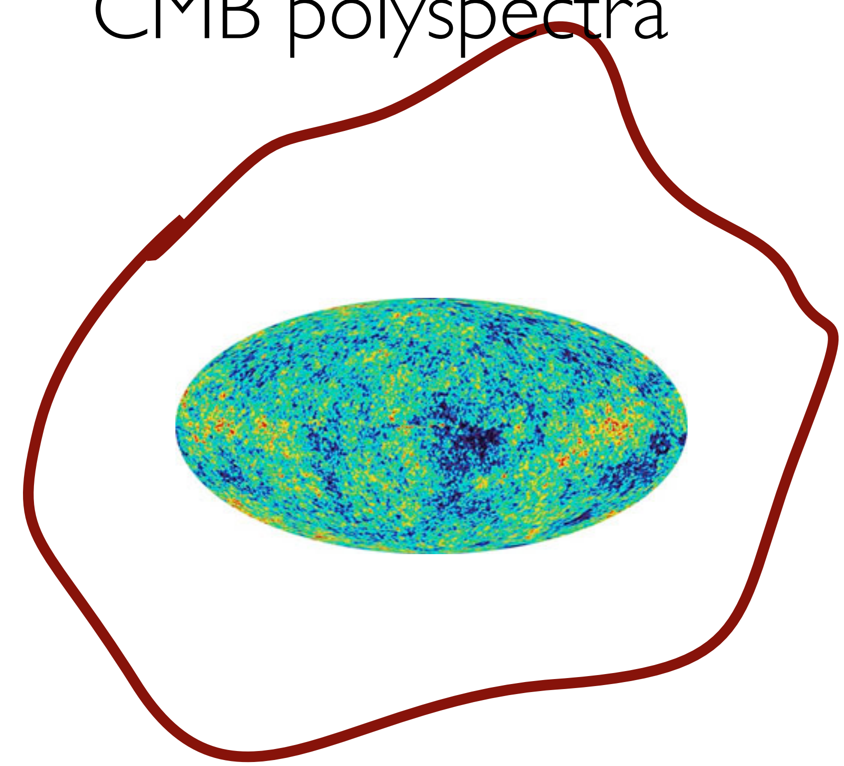


Expand model with primordial modes

Mode transfer functions





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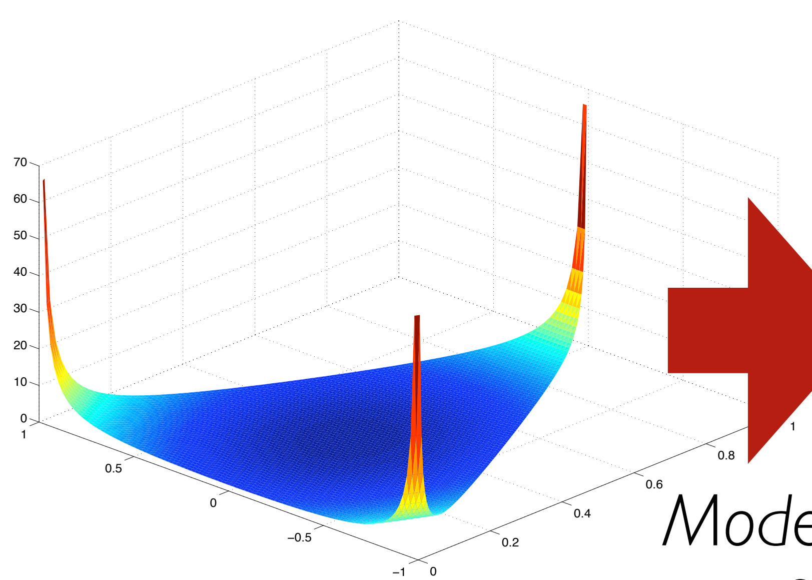

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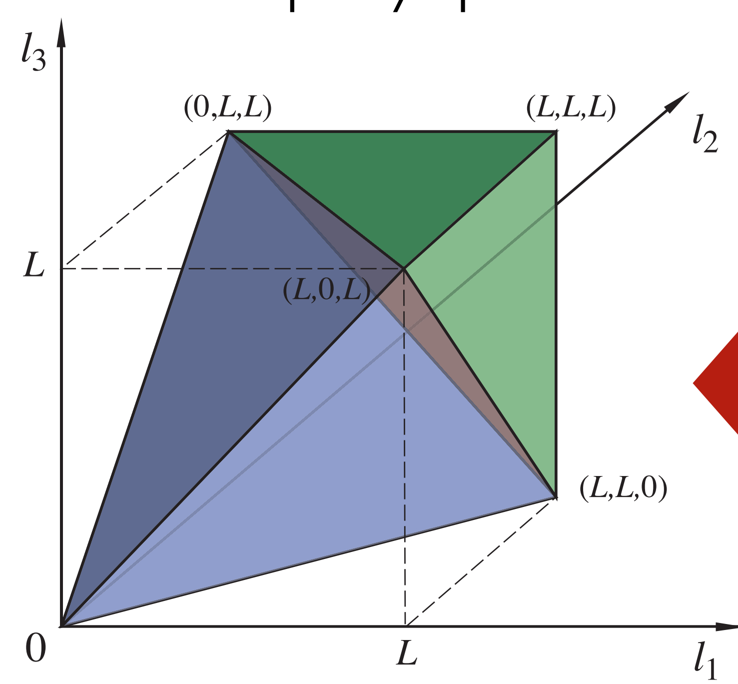

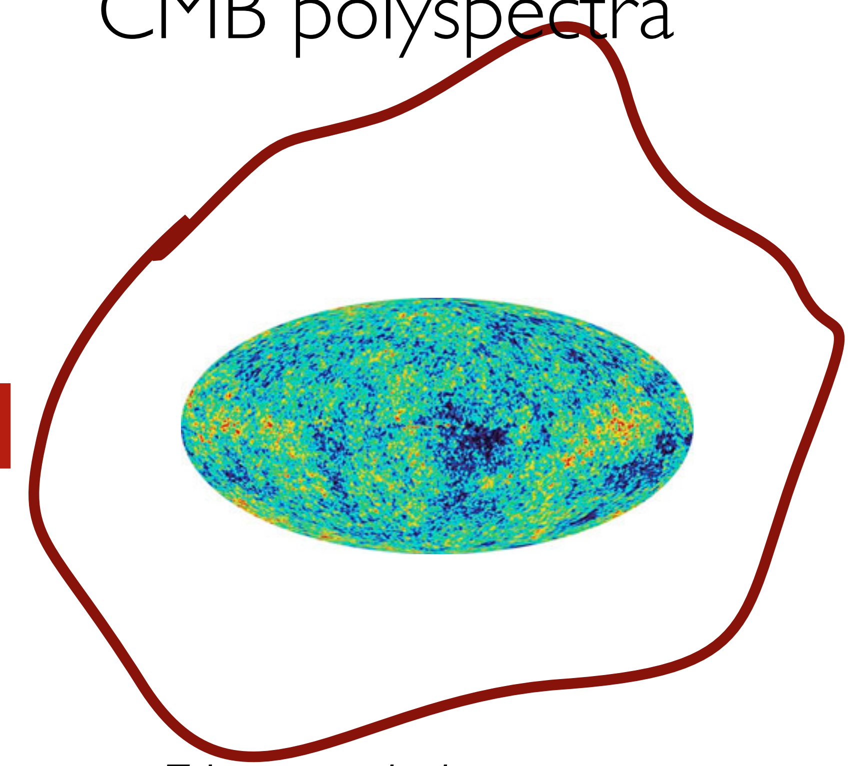

Filter with late-time projected modes

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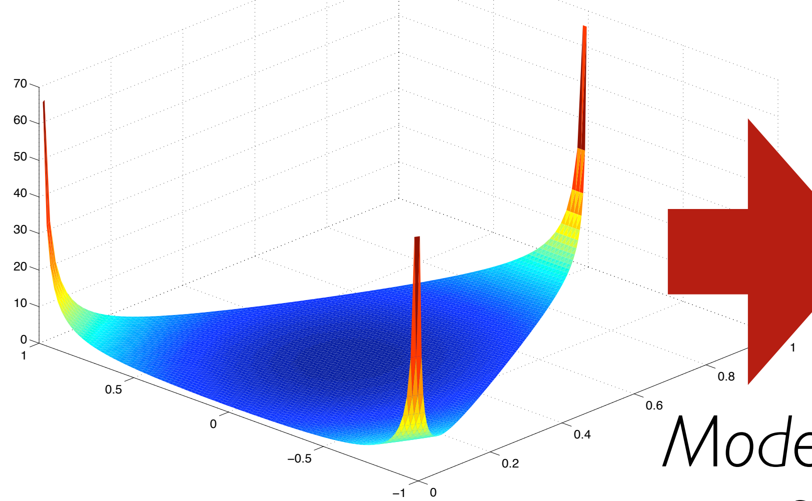

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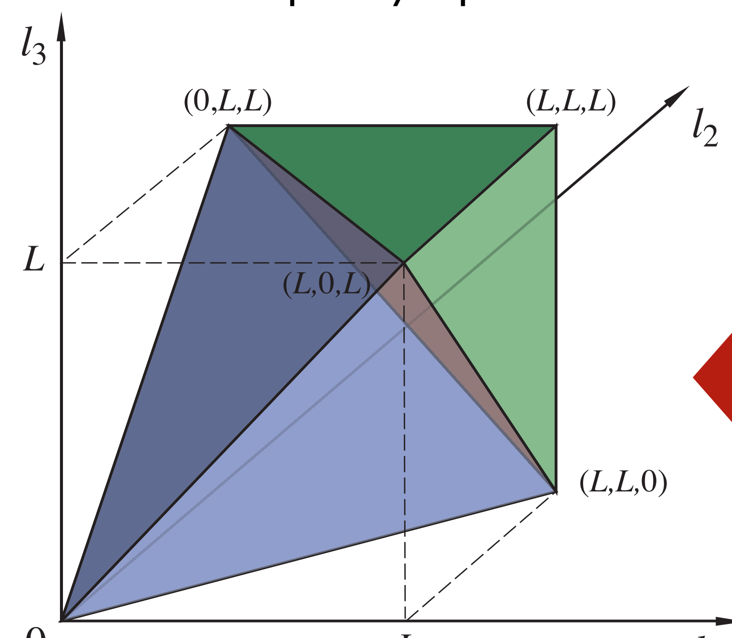
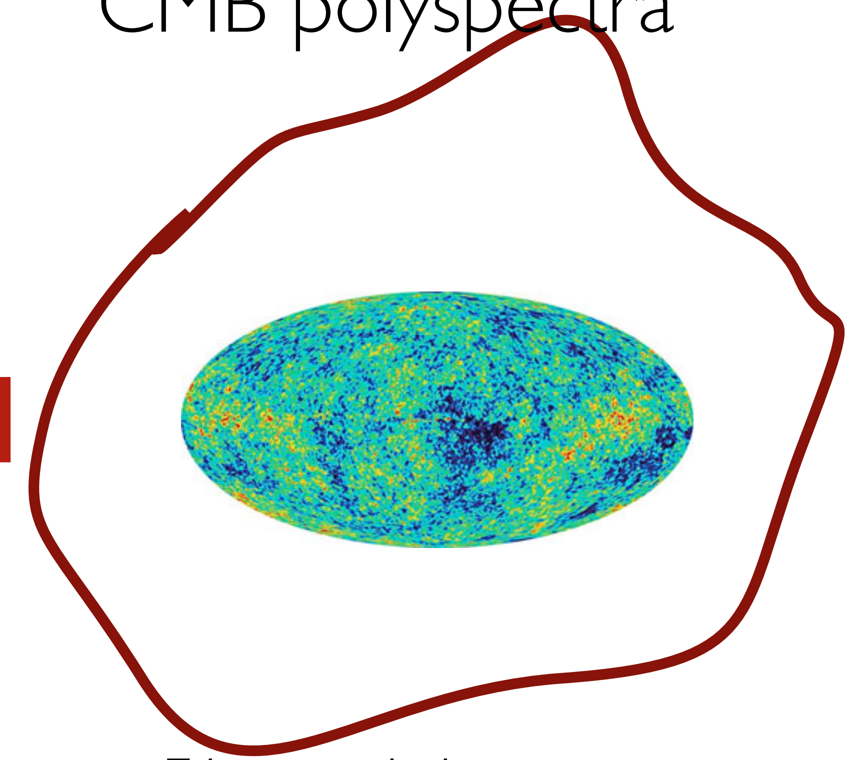


Modal estimator

$$\mathcal{E} = \frac{\sum_n \bar{\alpha}_n^R \bar{\beta}_n^R}{\sum_n \left(\bar{\alpha}_n^R\right)^2}$$



Filter with late-time projected modes

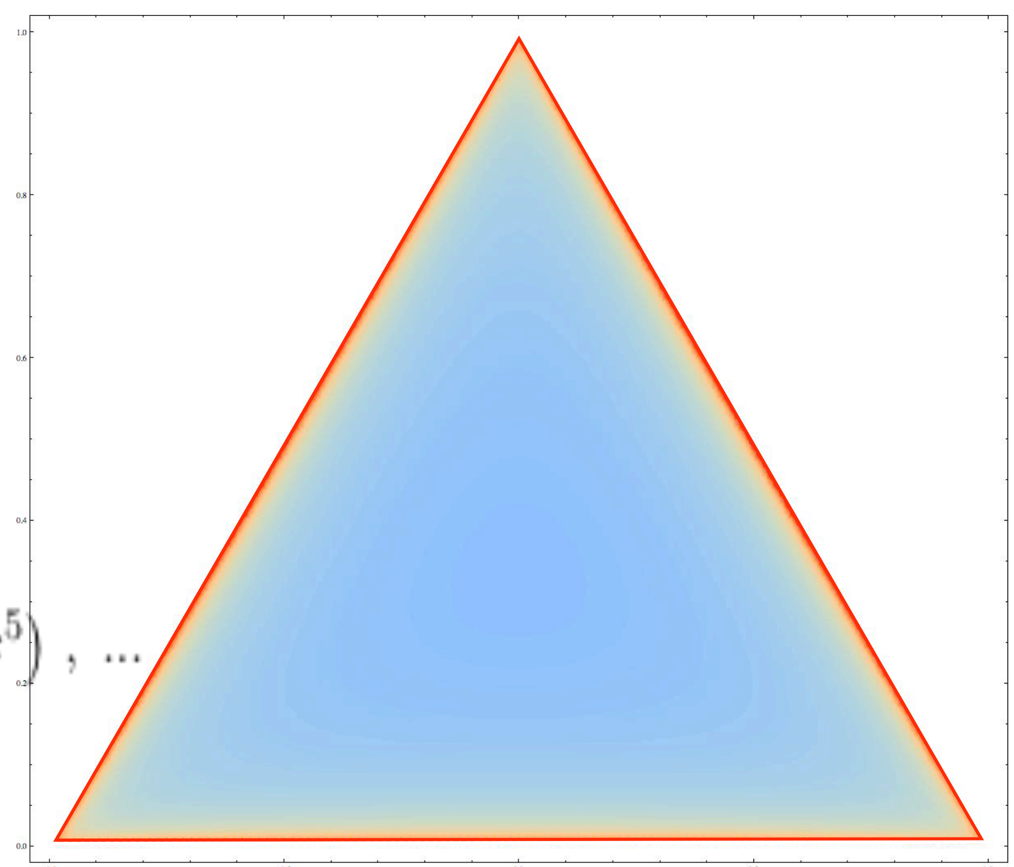
PRIMORDIAL MODES

Expand the shape using separable basis functions

$$S(k_1, k_2, k_3) = \sum_p \sum_r \sum_s \alpha_{prs}\, q_p(k_1)\, q_r(k_2)\, q_s(k_3)$$

Choose separable polynomials on the weighted tetrapyd

$$\begin{aligned}
q_0(x) &= \sqrt{2}\,,\\
q_1(x) &= 5.787\,(-\tfrac{7}{12} + x)\,,\\
q_2(x) &= 23.32\left(\tfrac{54}{215} - \tfrac{48}{43}x + x^2\right)\,,\\
q_3(x) &= 93.83\left(-0.09337 + 0.7642\,x - 1.631\,x^2 + x^3\right)\,,\\
q_4(x) &= 376.9\left(0.03192 - 0.4126\,x + 1.531\,x^2 - 2.139\,x^3 + x^4\right)\,,\\
q_5(x) &= 1512\left(-0.01033 + 0.1929\,x - 1.084\,x^2 + 2.549\,x^3 - 2.644\,x^4 + x^5\right)\,,\ \ldots
\end{aligned}$$



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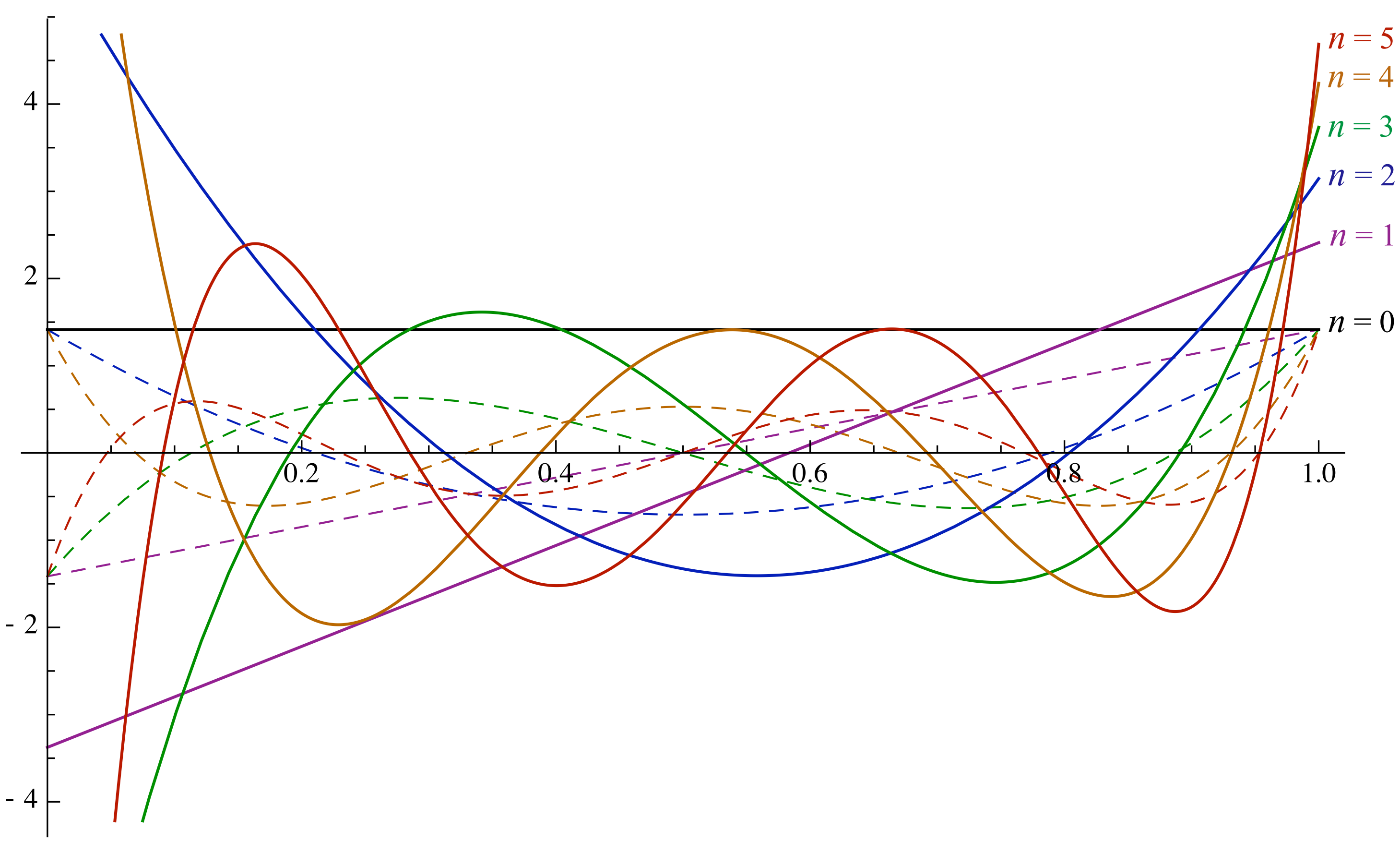
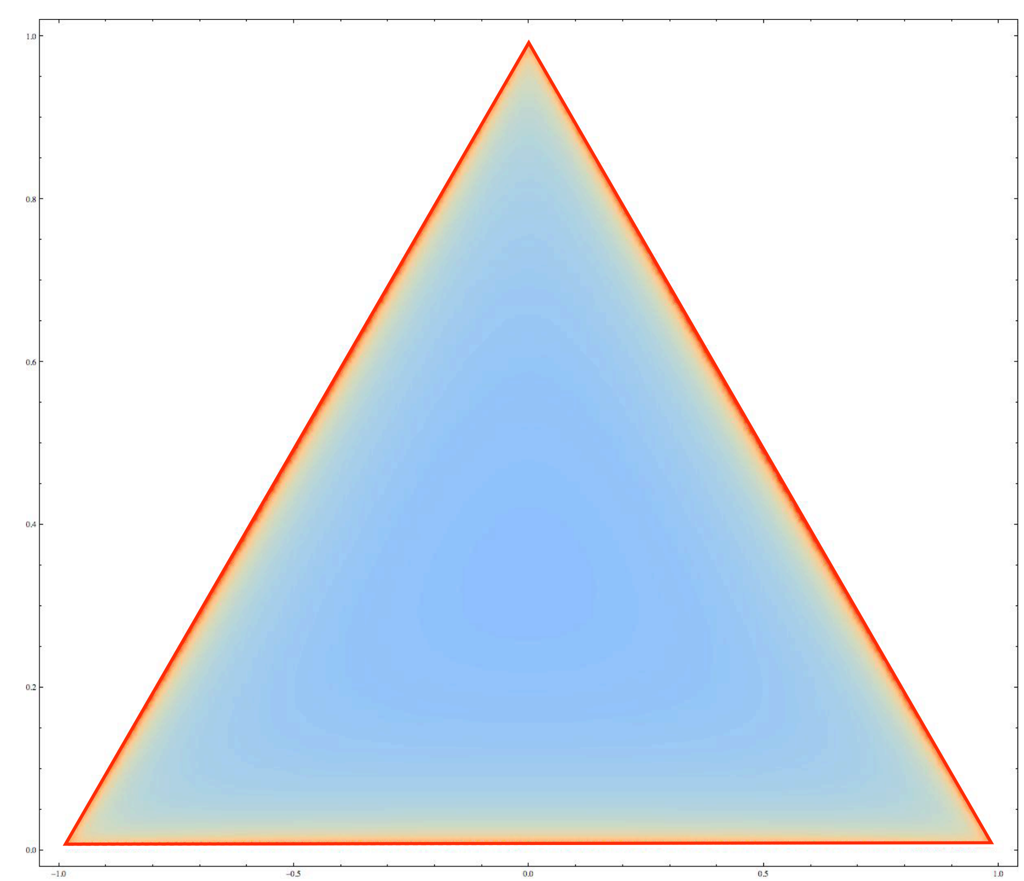


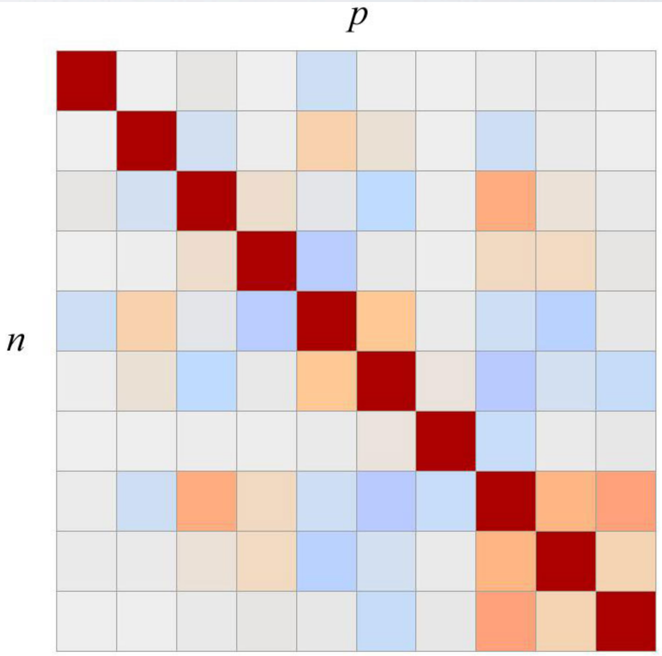


ORTHONORMAL BASIS

Next, as the shape function is symmetric in the three k, we create a symmetric product of the three polynomials of different orders which we label with n

$$\begin{aligned}\mathcal{Q}_n(x,y,z) = \tfrac{1}{6\mathcal{N}}\,[&q_p(x)q_r(y)q_s(z) + q_r(x)q_s(y)q_p(z) + q_s(x)q_p(y)q_r(z) \\ &+ q_p(x)q_s(y)q_r(z) + q_s(x)q_r(y)q_p(z) + q_r(x)q_p(y)q_s(z)] \\ \equiv q_{\{p}\, q_r\, q_{s\}} \quad &\text{with} \quad n \leftrightarrow \{prs\}\,,\end{aligned}$$

$$\begin{array}{llll} \underline{0 \rightarrow 000} & 4 \rightarrow 111 & 8 \rightarrow 022 & 12 \rightarrow 113 \\ \underline{1 \rightarrow 001} & 5 \rightarrow 012 & 9 \rightarrow 013 & 13 \rightarrow 023 \\ 2 \rightarrow 011 & \underline{6 \rightarrow 003} & \underline{10 \rightarrow 004} & 14 \rightarrow 014 \\ \underline{3 \rightarrow 002} & 7 \rightarrow 112 & 11 \rightarrow 122 & \underline{15 \rightarrow 005} \;\cdots \end{array}$$



Finally we orthonormalise the Q to create our basis R

$$\langle \mathcal{R}_n,\, \mathcal{R}_p \rangle = \delta_{np} \qquad \mathcal{R}_m = \sum_{p=0}^{m} \lambda_{mp}\mathcal{Q}_p \quad \text{for} \quad m,p \leq n$$

Orthonormal basis

Now we need to calculate λ_{nm}

$$\langle R_n R_m \rangle = \lambda_{nr} \lambda_{ms} \langle Q_r Q_s \rangle$$
$$\langle Q_r Q_s \rangle = \gamma_{rs}$$
$$I = \lambda \gamma \lambda^T$$

And rearranging noting that λ_{nm} is lower triangular we find it is the inverse of the Cholesky decomposition of the γ_{rs} matrix

$$\gamma = \lambda^{-1} {\lambda^{-1}}^T$$

CMB DECOMPOSITION

What should we decompose? To get the best correlation we should try to expand the reduced bispectrum with a separable weight which is as close to the square root of the estimator as possible

$$\frac{v_{l_1} v_{l_2} v_{l_3}}{\sqrt{C_{l_1} C_{l_2} C_{l_3}}} b_{l_1 l_2 l_3} = \sum_n \bar{\alpha}_n^{\mathcal{Q}} \overline{\mathcal{Q}}_n$$

$$v_l = (2l+1)^{1/6} \qquad \sqrt{\frac{(2l_1+1)(2l_2+1)(2l_3+1)}{4\pi}} \begin{pmatrix} l_1 & l_2 & l_3 \\ 0 & 0 & 0 \end{pmatrix}$$

Orthonormal CMB basis

So what is the inner product now?

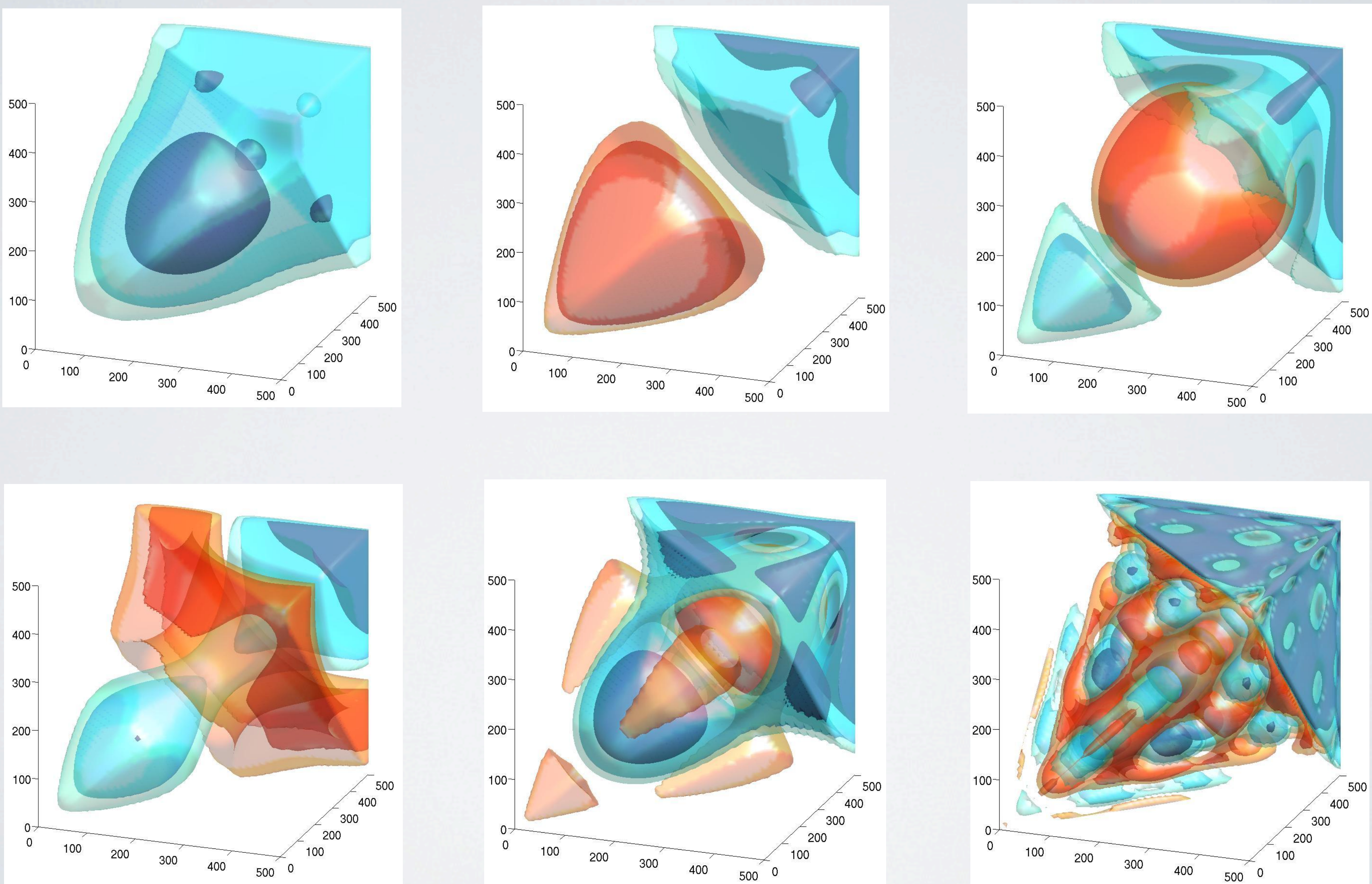
$$\mathcal{E} = \sum_{l_i m_i} \frac{\int Y_{l_1 m_2} Y_{l_2 m_2} Y_{l_3 m_3} b_{l_1 l_2 l_3} a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3}}{C_{l_1} C_{l_2} C_{l_3}}$$

$$= \sum_{l_i m_i} \left(\int Y_{l_1 m_2} Y_{l_2 m_2} Y_{l_3 m_3} \right)^2 \frac{b^{th}_{l_1 l_2 l_3} b^{obs}_{l_1 l_2 l_3}}{C_{l_1} C_{l_2} C_{l_3}}$$

$$= \sum_{l_i m_i} \left(\int \frac{Y_{l_1 m_2} Y_{l_2 m_2} Y_{l_3 m_3}}{v_{l_1} v_{l_2} v_{l_3}} \right)^2 \frac{v_{l_1} v_{l_2} v_{l_3} b^{th}_{l_1 l_2 l_3}}{\sqrt{C_{l_1} C_{l_2} C_{l_3}}} \frac{v_{l_1} v_{l_2} v_{l_3} b^{obs}_{l_1 l_2 l_3}}{\sqrt{C_{l_1} C_{l_2} C_{l_3}}}$$

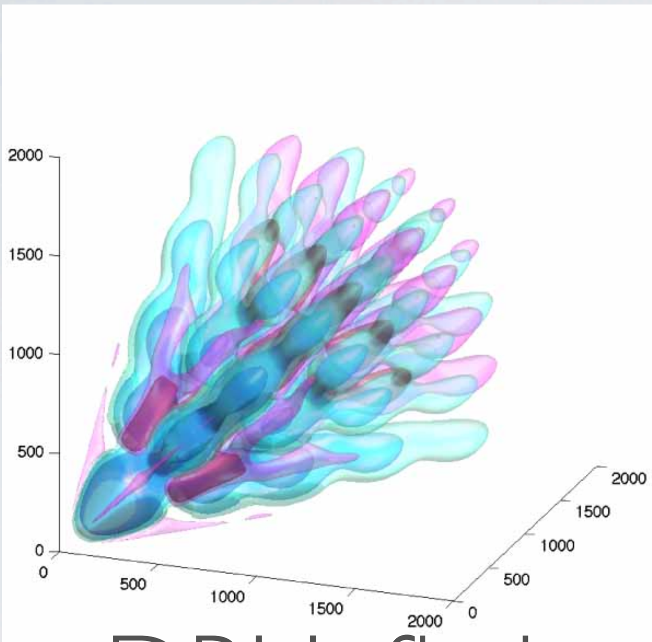
And we choose v to make the weight as flat as possible

CMB DECOMPOSITION

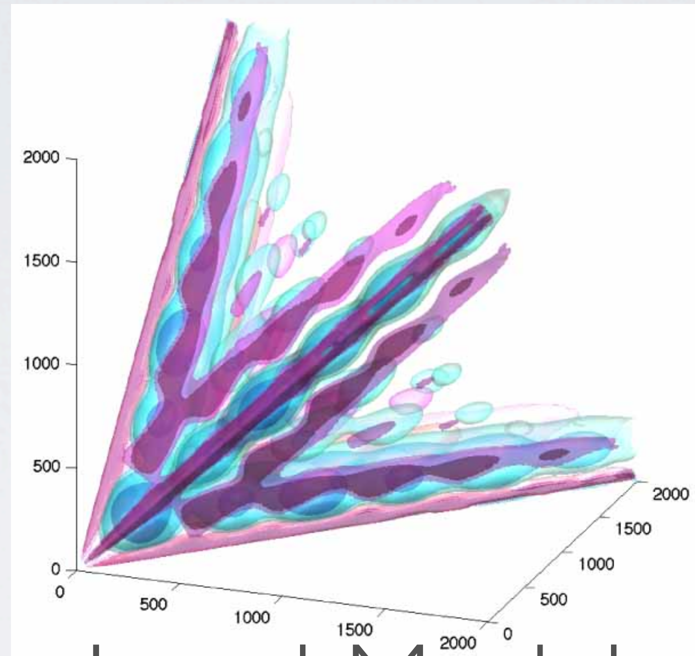


Applicable to inflation, defects, secondaries

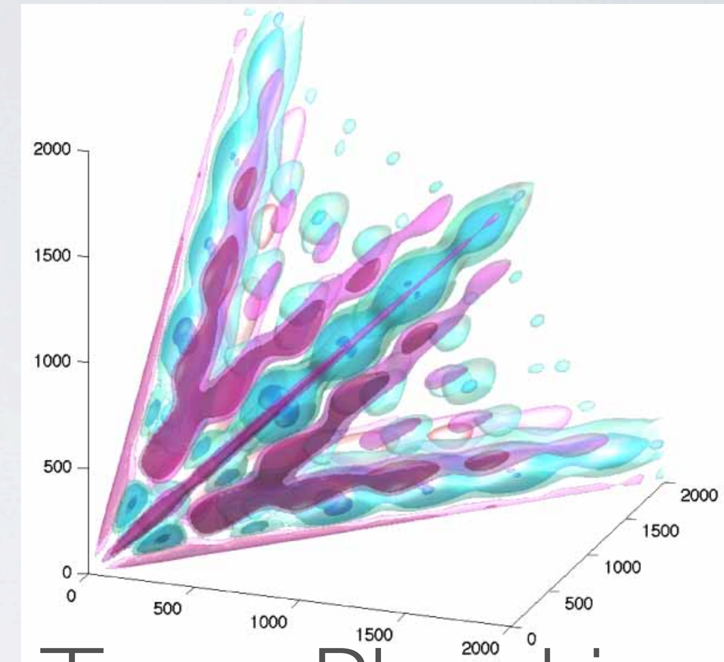
Planck resolution CMB bispectra (multipoles l_1, l_2, l_3)



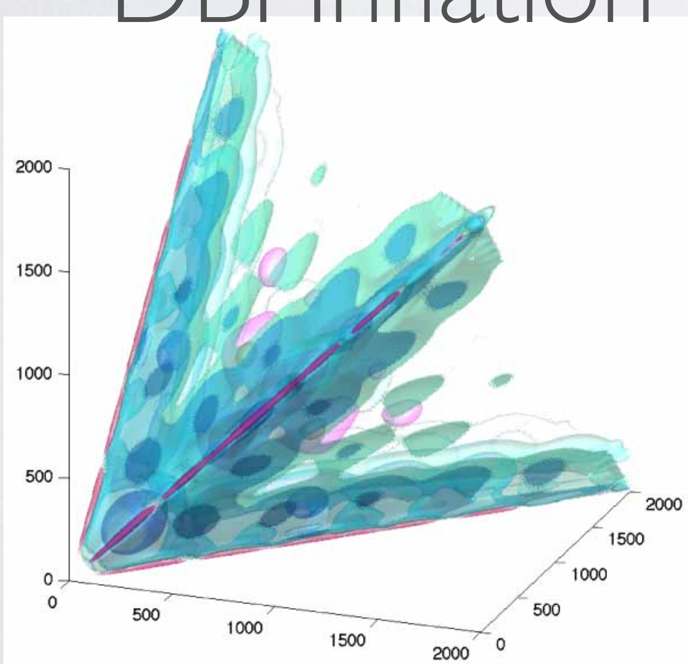
DBI Inflation



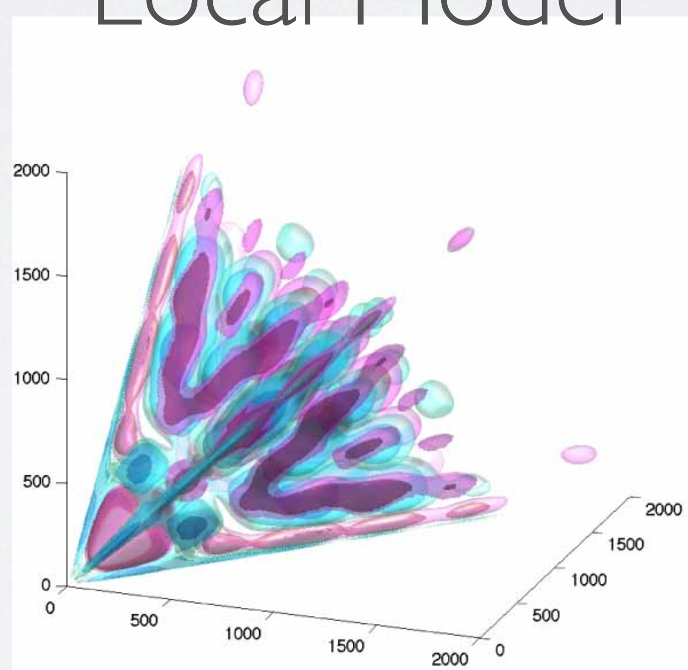
Local Model



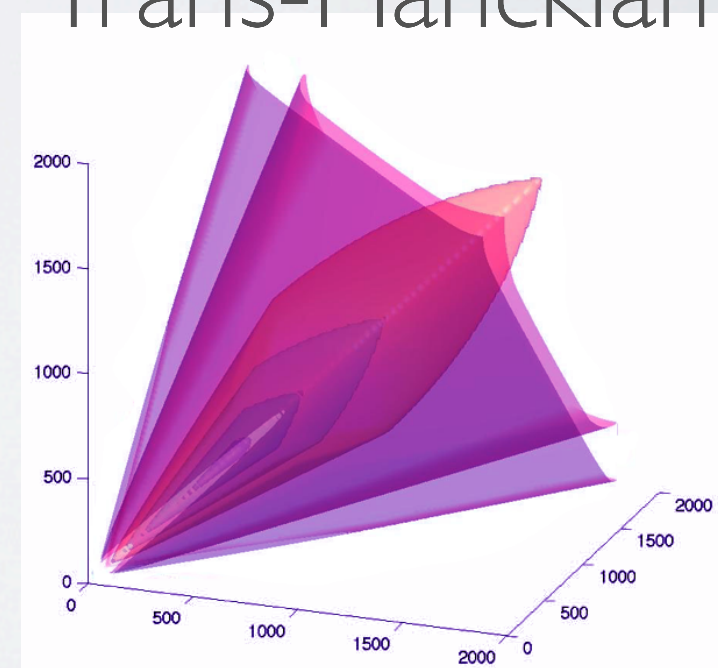
Trans-Planckian



Warm inflation



Feature Model



Cosmic strings

Primordial to CMB basis

Use transfer functions *once* to project forward primordial modes so we calculate

$$\Gamma_{nm} = \left\langle \bar{Q}^n \frac{vvv\tilde{Q}^m}{\sqrt{CCC}} \right\rangle$$

Then we can transform between the primordial and CMB expansions

$$\bar{\alpha}^Q = \bar{\gamma}^{-1}\Gamma\alpha^Q$$

CMB DECOMPOSITION

$$\mathcal{E} = \sum_{l_i,m_i} \sum_{n\leftrightarrow prs} \bar{\alpha}_n^{\mathcal{Q}} \bar{q}_{\{p} \bar{q}_r \bar{q}_{s\}} \int d^2\hat{\mathbf{n}}\, Y_{l_2m_2}(\hat{\mathbf{n}}) Y_{l_1m_1}(\hat{\mathbf{n}})\, Y_{l_3m_3}(\hat{\mathbf{n}})\, \frac{a_{l_1m_1}a_{l_2m_2}a_{l_3m_3}}{v_{l_1}v_{l_2}v_{l_3}\sqrt{C_{l_1}C_{l_2}C_{l_3}}}$$

$$= \sum_{n\leftrightarrow prs} \bar{\alpha}_n^{\mathcal{Q}} \int d^2\hat{\mathbf{n}} \left(\sum_{l_1,m_1} \bar{q}_{\{p} \frac{a_{l_1m_1}Y_{l_1m_1}}{v_{l_1}\sqrt{C_{l_1}}} \right) \left(\sum_{l_2,m_2} \bar{q}_r \frac{a_{l_2m_2}Y_{l_2m_2}}{v_{l_2}\sqrt{C_{l_2}}} \right) \left(\sum_{l_3,m_3} \bar{q}_{s\}} \frac{a_{l_3m_3}Y_{l_3m_3}}{v_{l_3}\sqrt{C_{l_3}}} \right)$$

$$\bar{M}_p(\hat{\mathbf{n}}) = \sum_{lm} q_p(l) \frac{a_{lm}}{v_l\sqrt{C_l}} Y_{lm}(\hat{\mathbf{n}})$$

$$\bar{\mathcal{M}}_n(\hat{\mathbf{n}}) = \bar{M}_p(\hat{\mathbf{n}})\bar{M}_r(\hat{\mathbf{n}})\bar{M}_s(\hat{\mathbf{n}})$$

$$\beta_n = \int d^2\hat{\mathbf{n}} \mathcal{M}_n(\hat{\mathbf{n}})$$

$$\mathcal{E} = \frac{1}{N} \sum_{n=0}^{n_{\max}} \bar{\alpha}_n^{\mathcal{Q}} \bar{\beta}_n^{\mathcal{Q}}$$

Now the projection is in alpha rather than beta

CMB DECOMPOSITION

$$\mathcal{E} = \sum_{l_i,m_i} \sum_{n\leftrightarrow prs} \bar{\alpha}_n^{\mathcal{Q}} \bar{q}_{\{p}\bar{q}_r\bar{q}_{s\}} \int d^2\hat{\mathbf{n}}\, Y_{l_2m_2}(\hat{\mathbf{n}}) Y_{l_1m_1}(\hat{\mathbf{n}})\, Y_{l_3m_3}(\hat{\mathbf{n}})\, \frac{a_{l_1m_1}a_{l_2m_2}a_{l_3m_3}}{v_{l_1}v_{l_2}v_{l_3}\sqrt{C_{l_1}C_{l_2}C_{l_3}}}$$

$$= \sum_{n\leftrightarrow prs} \bar{\alpha}_n^{\mathcal{Q}} \int d^2\hat{\mathbf{n}} \left(\sum_{l_1,m_1} \bar{q}_{\{p} \frac{a_{l_1m_1}Y_{l_1m_1}}{v_{l_1}\sqrt{C_{l_1}}} \right) \left(\sum_{l_2,m_2} \bar{q}_r \frac{a_{l_2m_2}Y_{l_2m_2}}{v_{l_2}\sqrt{C_{l_2}}} \right) \left(\sum_{l_3,m_3} \bar{q}_{s\}} \frac{a_{l_3m_3}Y_{l_3m_3}}{v_{l_3}\sqrt{C_{l_3}}} \right)$$

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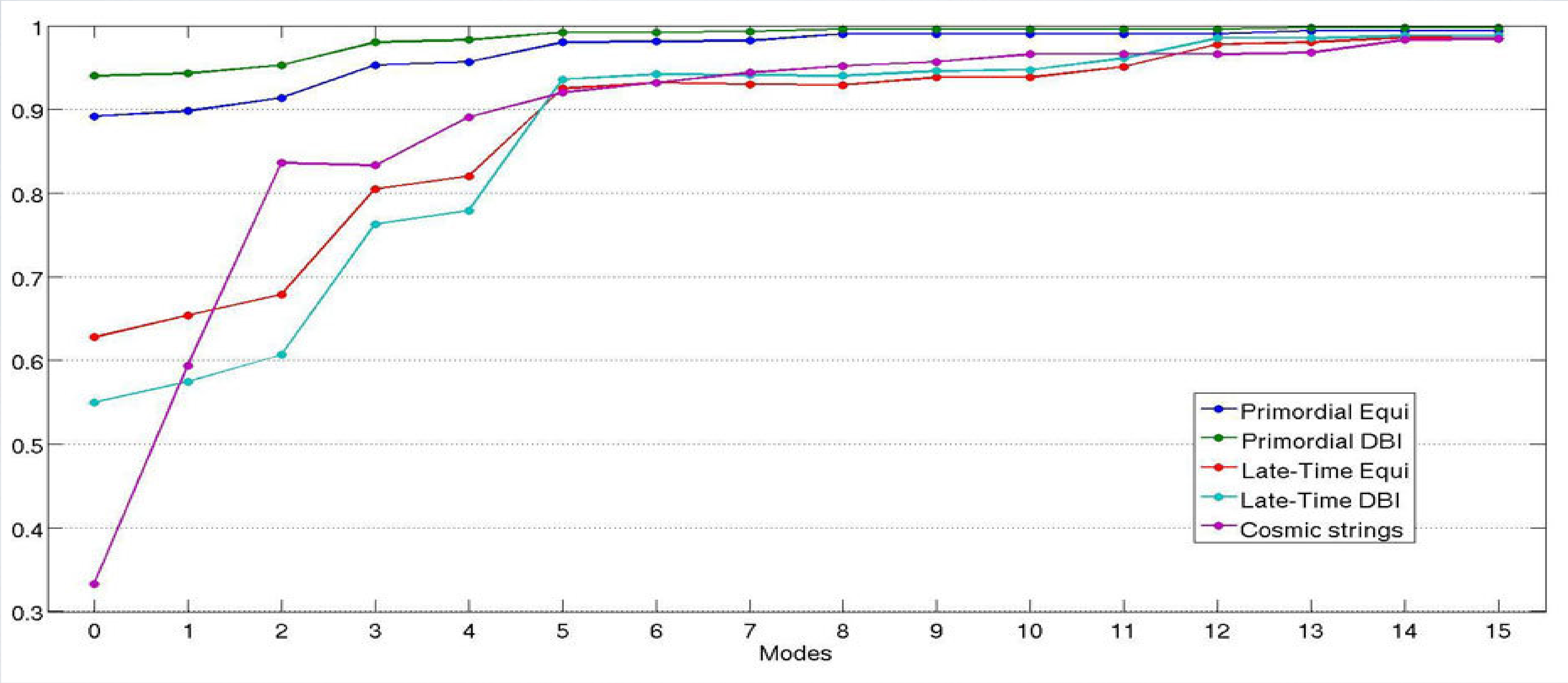
$$\mathcal{E} = \frac{1}{N} \sum_{n=0}^{n_{\max}} \bar{\alpha}_n^{\mathcal{Q}} \bar{\beta}_n^{\mathcal{Q}}$$

Now the projection is in alpha rather than beta

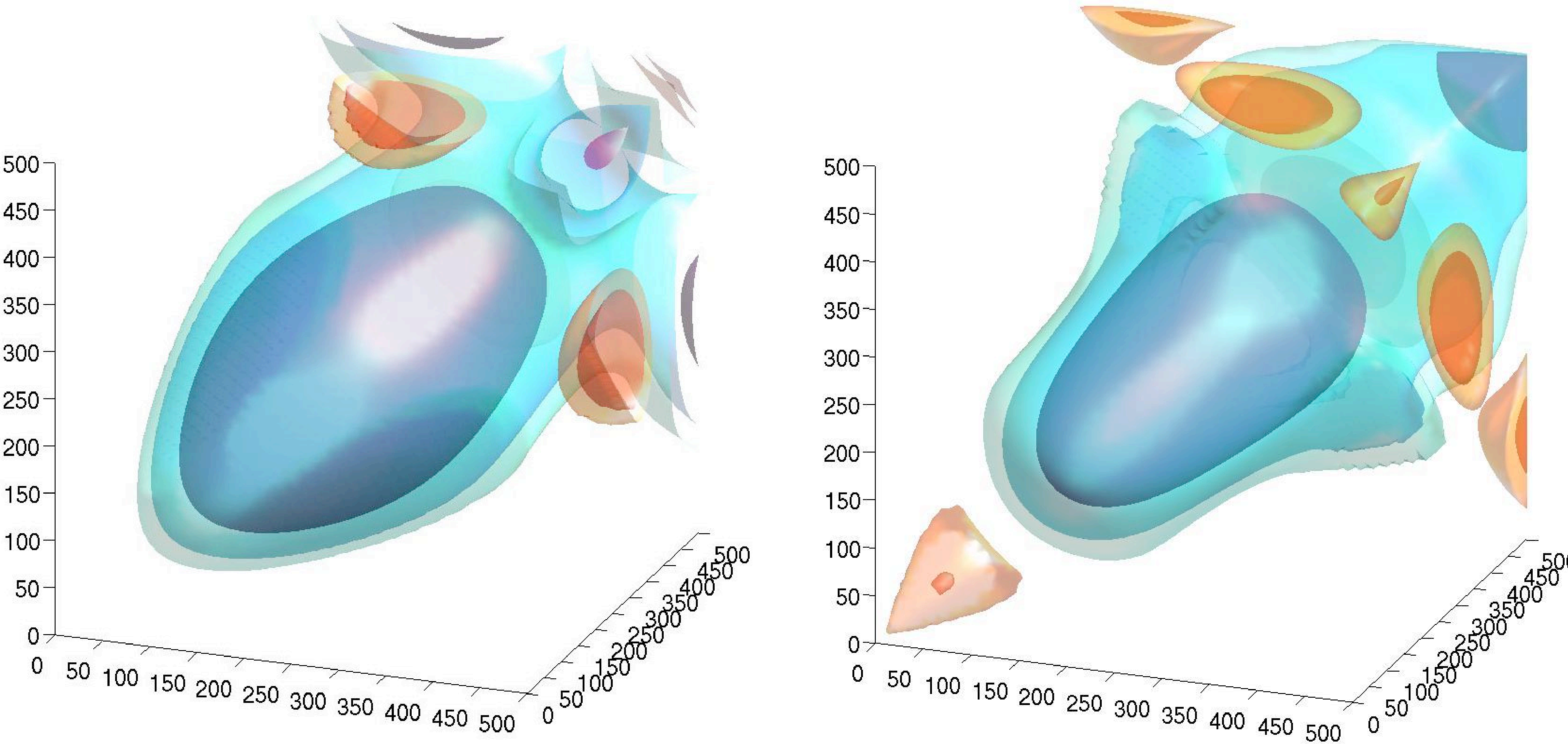
CMB DECOMPOSITION



Correlation of the separable approximation to the original bispectra, both primordial and CMB

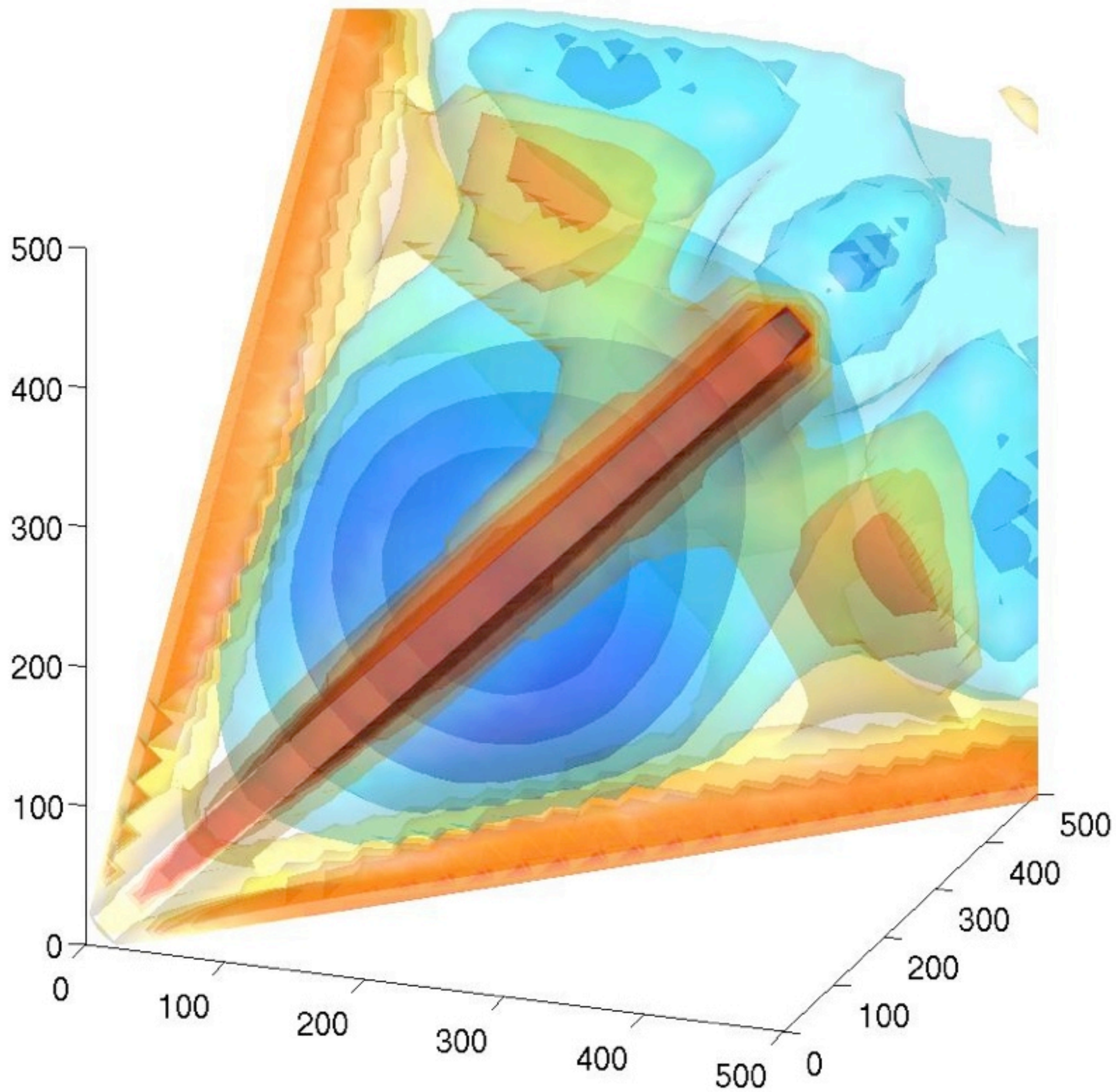
Bispectrum reconstruction

Recovery of 3sigma equil & local bispectrum signal from simulated maps with WMAP noise, beam and KQ75 mask

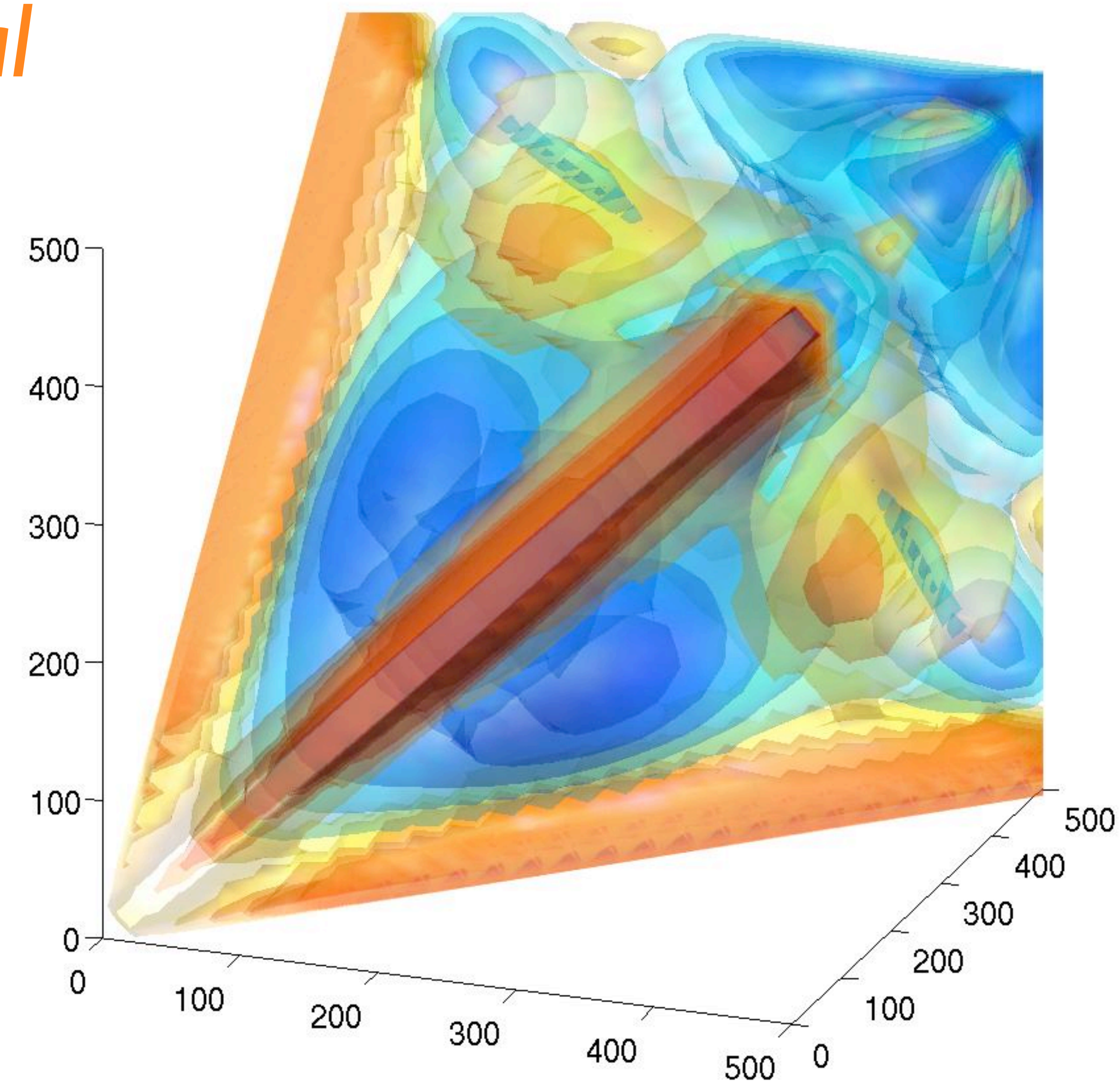


Bispectrum reconstruction

Recovery of 3sigma equil & local bispectrum signal from simulated maps with WMAP noise, beam and KQ75 mask



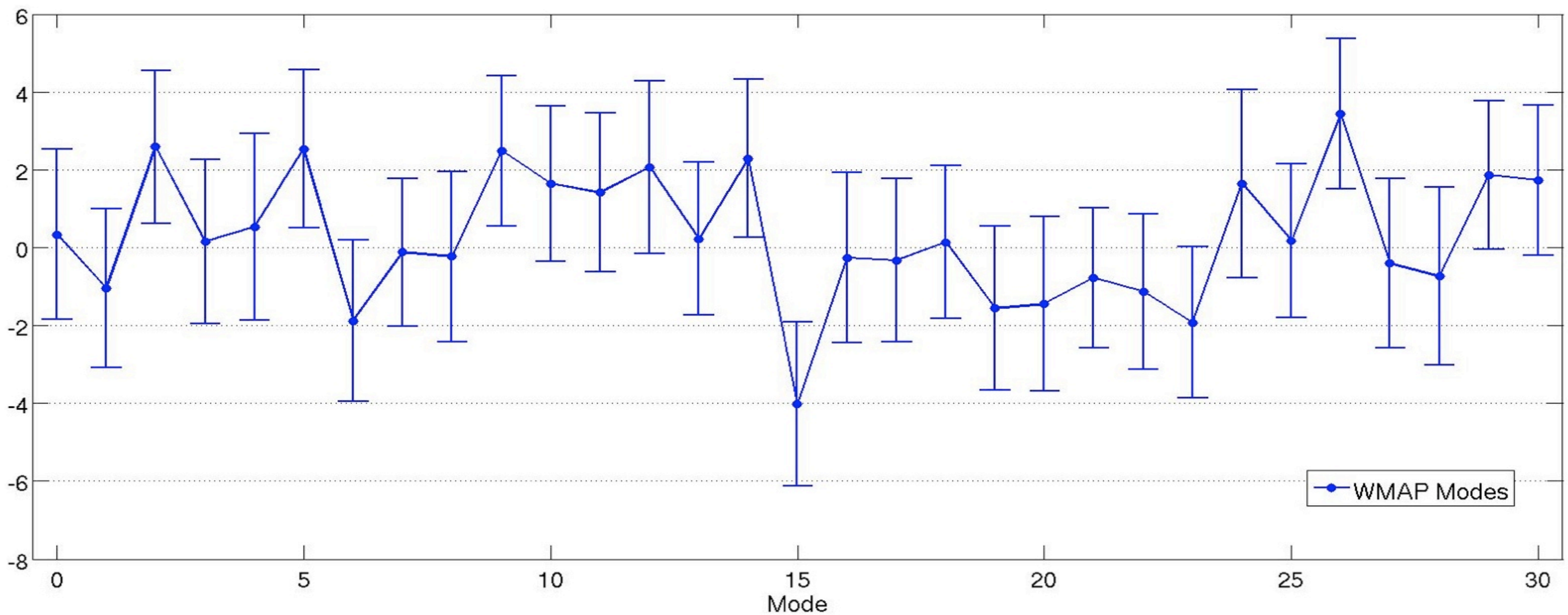
Local



WMAP mode decomposition

Orthonormal coefficients from preliminary WMAP5 analysis

β_n



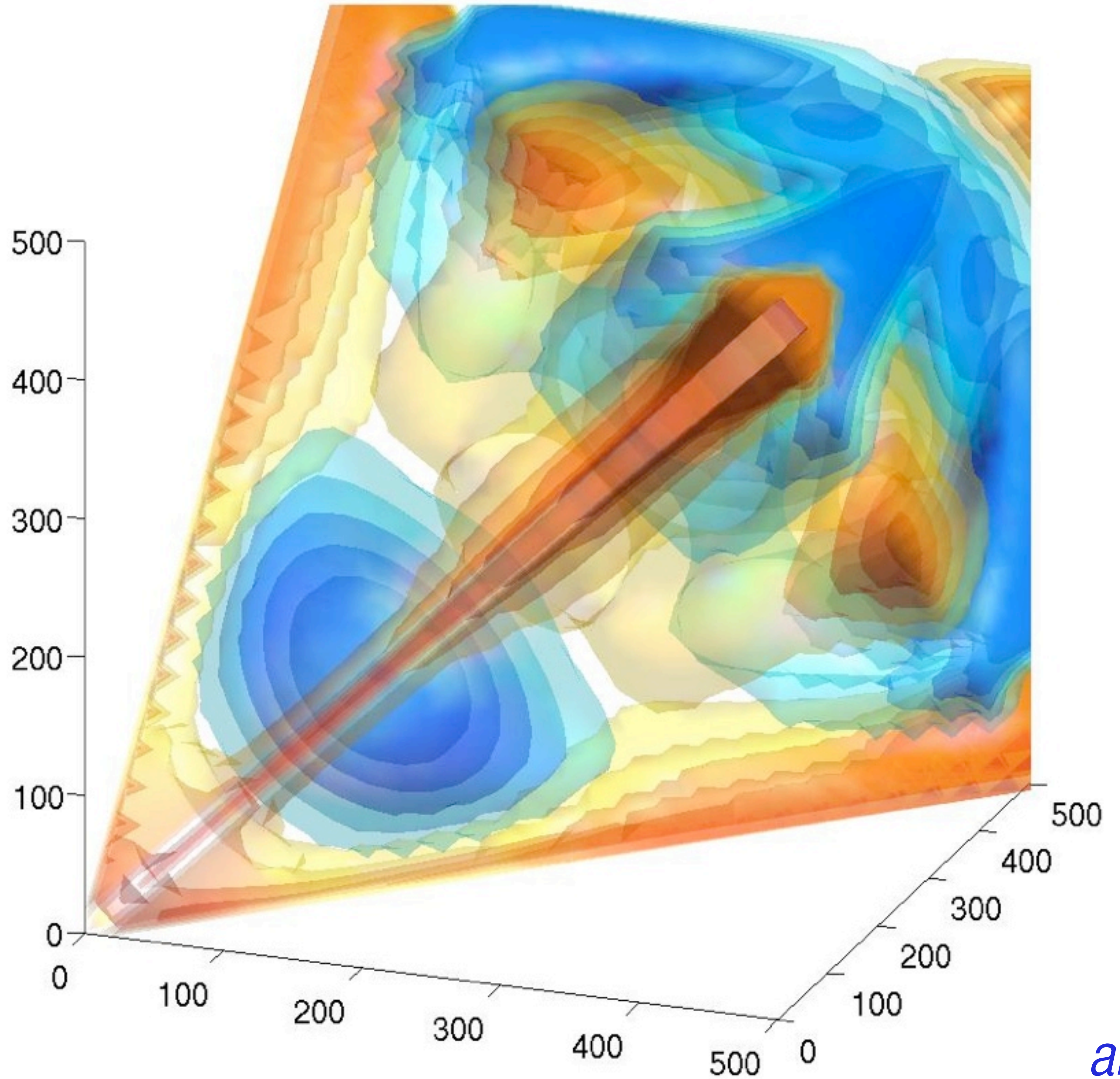
Eigenmode n

Note: Late-time general method see all bispectrum contributions (unlike specific primordial filters)

Larger than expected foreground or other contamination, but inhomogeneous signal successfully subtracted by linear term

The Bispectrum of the Universe

WMAP5 bispectrum after noise subtraction



arXiv:1006.1642

Estimation

We have used these methods to constrain all scale invariant models and an oscillatory model for a selection of parameter space via the bispectrum

Model	$F_{\rm NL}$	$(f_{\rm NL})$
Constant	35.1 ± 27.4	(149.4 ± 116.8)
DBI	26.7 ± 26.5	(146.0 ± 144.5)
Equilateral	25.1 ± 26.4	(143.5 ± 151.2)
Flat (Smoothed)	35.4 ± 29.2	(18.1 ± 14.9)
Ghost	22.0 ± 26.3	(138.7 ± 165.4)
Local	54.4 ± 29.4	(54.4 ± 29.4)
Orthogonal	-16.3 ± 27.3	(-79.4 ± 133.3)
Single	28.8 ± 26.6	(142.1 ± 131.3)
Warm	24.2 ± 27.3	(94.7 ± 106.8)

Phase \ **Scale**	150	200	250	300	400	500	600	700
0	57 (30)	−52 (33)	−25 (32)	1 (30)	1 (27)	8 (26)	18 (25)	23 (25)
$\pi/8$	67 (36)	−26 (27)	−36 (30)	−6 (25)	−4 (26)	−2 (27)	12 (26)	20 (25)
$\pi/4$	68 (42)	−10 (29)	−43 (30)	−11 (21)	−7 (25)	−10 (27)	−1 (28)	13 (27)
$3\pi/8$	49 (46)	7 (34)	−42 (32)	−18 (24)	−9 (25)	−14 (26)	−13 (28)	−2 (28)
$\pi/2$	15 (46)	32 (41)	−30 (35)	−32 (34)	−10 (25)	−16 (25)	−18 (27)	−14 (28)
$5\pi/8$	−19 (42)	63 (46)	−15 (35)	−38 (43)	−11 (25)	−16 (25)	−20 (26)	−20 (27)
$3\pi/4$	−39 (35)	87 (48)	0 (35)	−25 (41)	−11 (26)	−15 (25)	−21 (25)	−23 (26)
$7\pi/8$	−48 (30)	81 (43)	13 (34)	−11 (35)	−7 (27)	−13 (25)	−20 (25)	−23 (25)

$$S^{feat}(k_1,k_2,k_3) = \frac{1}{N}\sin\left(2\pi\frac{k_1+k_2+k_3}{3k^*}+\Phi\right)$$

NORMALIZATION & BLIND NG SURVEY

$$F_{\rm NL} = \frac{1}{N\bar{N}_{\rm loc}} \sum_{l_i m_i} \mathcal{G}^{l_1 l_2 l_3}_{m_1 m_2 m_3} b_{l_1 l_2 l_3} \frac{a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3}}{C_{l_1} C_{l_2} C_{l_3}}$$

$$N^2 = \sum_{l_i} \frac{B^2_{l_1 l_2 l_3}}{C_{l_1} C_{l_2} C_{l_3}} \qquad \bar{N}^2_{\rm loc} = \sum_{l_i} \frac{B^{\rm loc\,(f_{\rm NL}=1)}_{l_1 l_2 l_3}{}^2}{C_{l_1} C_{l_2} C_{l_3}}$$

$$F_{\rm NL}^2 = \frac{1}{N} \sum_n \bar{\alpha}^{\mathcal{R}}_n{}^2$$

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NORMALIZATION & BLIND NG SURVEY

$$F_{\rm NL} = \frac{1}{N\bar{N}_{\rm loc}} \sum_{l_i m_i} \mathcal{G}^{l_1 l_2 l_3}_{m_1 m_2 m_3} b_{l_1 l_2 l_3} \frac{a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3}}{C_{l_1} C_{l_2} C_{l_3}}$$

$$N^2 = \sum_{l_i} \frac{B^2_{l_1 l_2 l_3}}{C_{l_1} C_{l_2} C_{l_3}} \qquad \bar{N}^2_{\rm loc} = \sum_{l_i} \frac{B^{{\rm loc}\,(f_{\rm NL}=1)^2}_{l_1 l_2 l_3}}{C_{l_1} C_{l_2} C_{l_3}}$$

$$F_{\rm NL}^2 = \frac{1}{N} \sum_n \bar{\alpha}_n^{\mathcal{R}\,2}$$

We live in a Gaussian Universe

NORMALIZATION & BLIND NG SURVEY

$$F_{\rm NL} = \frac{1}{N\bar{N}_{\rm loc}} \sum_{l_i m_i} \mathcal{G}^{l_1 l_2 l_3}_{m_1 m_2 m_3} b_{l_1 l_2 l_3} \frac{a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3}}{C_{l_1} C_{l_2} C_{l_3}}$$

$$N^2 = \sum_{l_i} \frac{B^2_{l_1 l_2 l_3}}{C_{l_1} C_{l_2} C_{l_3}} \qquad \bar{N}^2_{\rm loc} = \sum_{l_i} \frac{B^{{\rm loc}\,(f_{\rm NL}=1)^2}_{l_1 l_2 l_3}}{C_{l_1} C_{l_2} C_{l_3}}$$

$$F^2_{\rm NL} = \frac{1}{N} \sum_n \bar{\alpha}^{\mathcal{R}\,2}_n$$

We live in a Gaussian Universe ... for the moment!

Trispectrum estimation

- *Easiest to tackle the non-diagonal case (most models)*

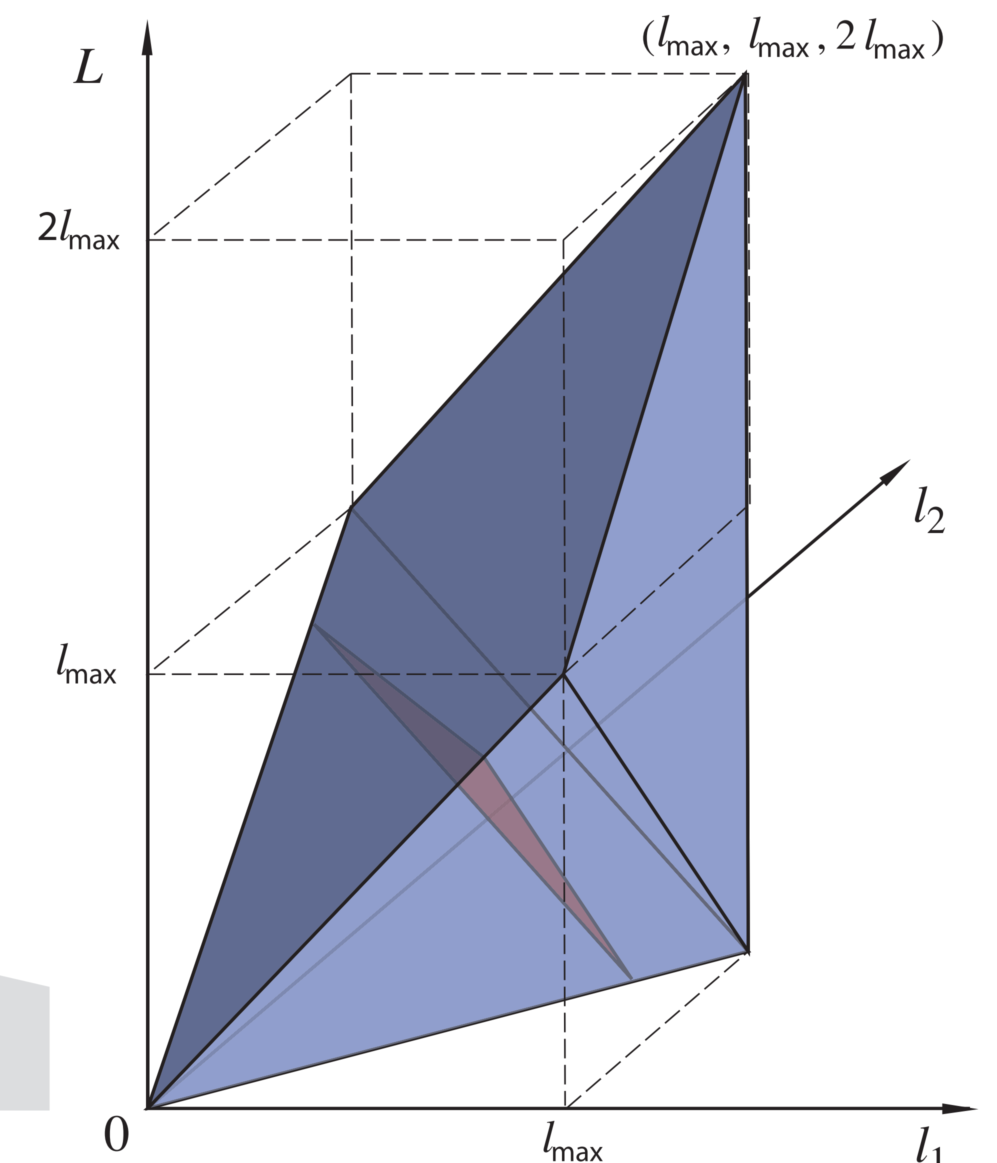
$$\langle \Phi(\mathbf{k}_1)\Phi(\mathbf{k}_2)\Phi(\mathbf{k}_3\Phi(\mathbf{k}_4))\rangle = (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3+\mathbf{k}_4)T(k_1,k_2,k_3,k_4)$$

$$\langle a_{l_1m_1}a_{l_2m_2}a_{l_3m_3}a_{l_4m_4}\rangle = \left(\int d^2\hat{n}Y_{l_1m_1}(\hat{\mathbf{n}})Y_{l_2m_2}(\hat{\mathbf{n}})Y_{l_3m_3}(\hat{\mathbf{n}})Y_{l_4m_4}(\hat{\mathbf{n}})\right) t_{l_1l_2l_3l_4}$$

Regan, EPS & JRF, arXiv:1004.2915

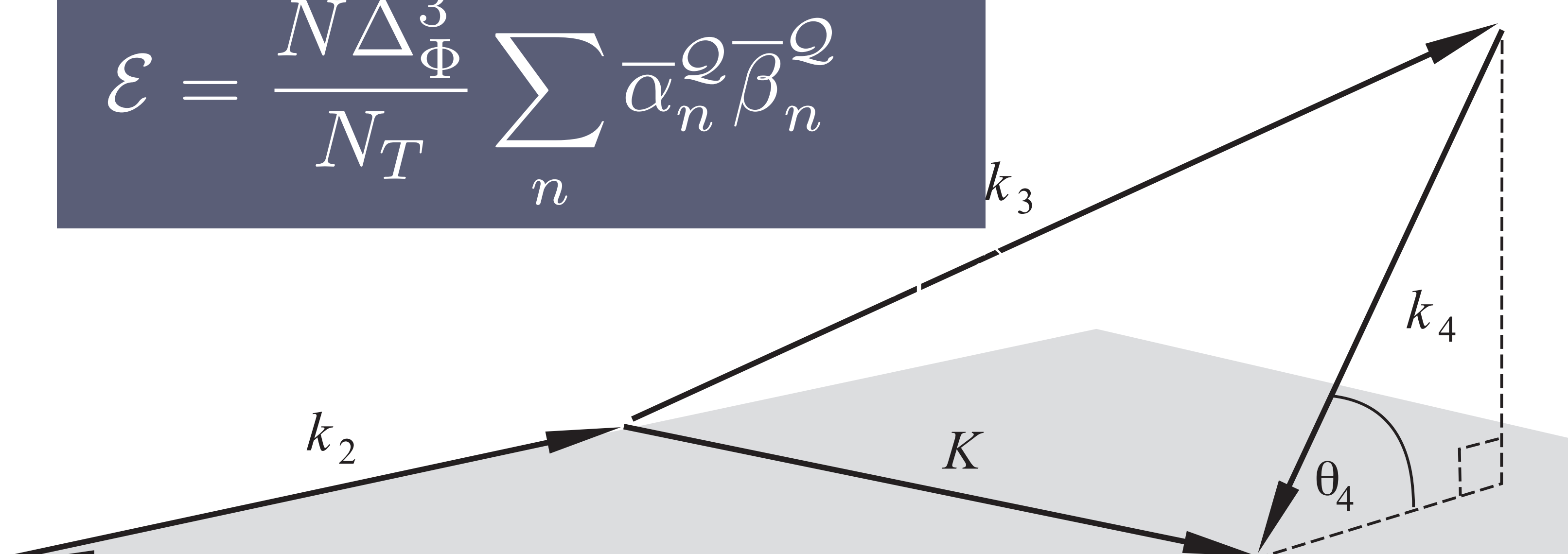
- *We can expand in polynomials*

$$\frac{v_{l_1}v_{l_2}v_{l_3}v_{l_4}v_L}{\sqrt{C_{l_1}C_{l_2}C_{l_3}C_{l_4}}} t^{l_1l_2}_{l_3l_4}(L) = \sum_m \overline{\alpha}^{\mathcal{Q}}_m \overline{\mathcal{Q}}_m$$



- *The modal estimator becomes*

$$\mathcal{E} = \frac{N\Delta^3_\Phi}{N_T}\sum_n \overline{\alpha}^{\mathcal{Q}}_n \overline{\beta}^{\mathcal{Q}}_n$$



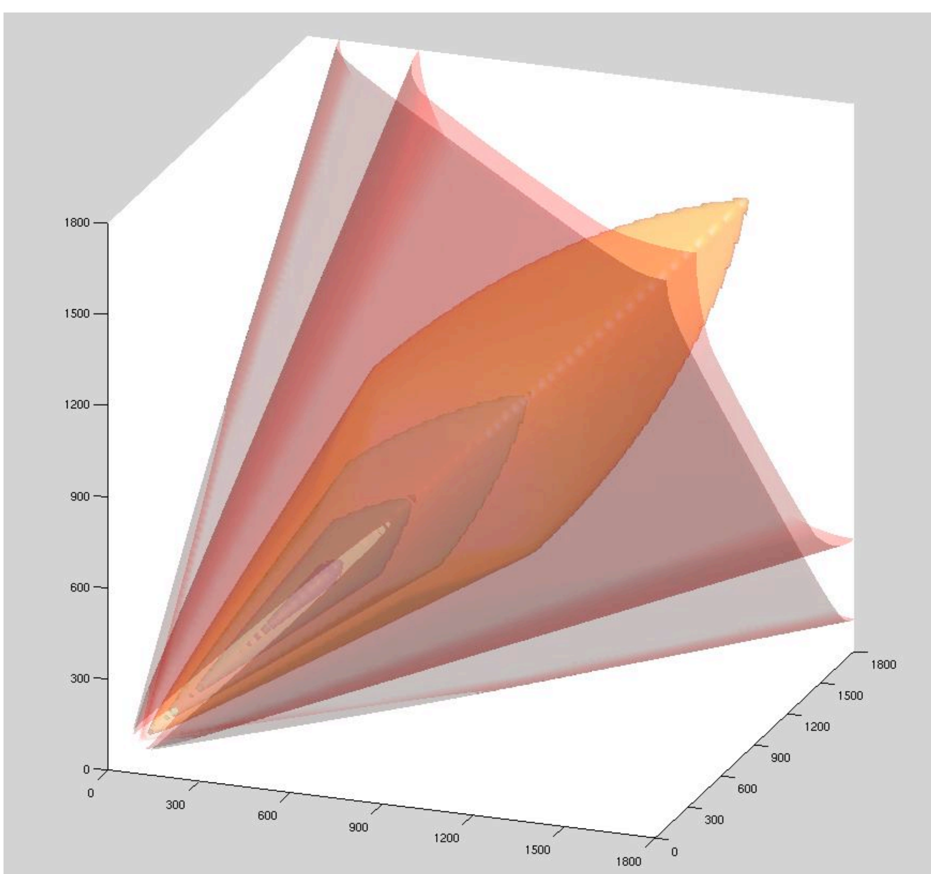
String Bispectrum & Trispectrum

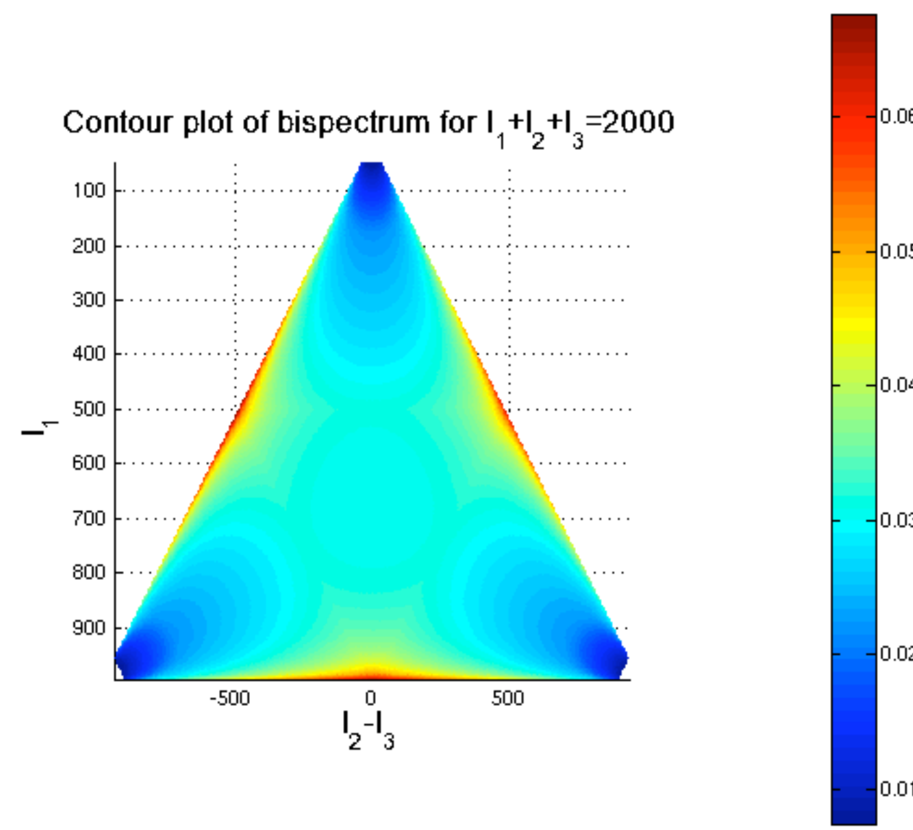
- *Late-time gravitational effect - integrated after decoupling*

$$b^{\text{string}}_{\ell_1\ell_2\ell_3} = \frac{A}{(\zeta\ell_1\ell_2\ell_3)^2}\left[(\ell_3^2-\ell_1^2-\ell_2^2)\left(\frac{L}{2\ell_3}+\frac{\ell_3}{50L}\right)\sqrt{\frac{\ell_*}{500}}\,\mathrm{erf}(0.3\zeta\ell_3) \;+\; 2\text{ perm.}\right], \qquad (\ell \le 2000)\,,$$

where $\ell_{min} = \min(\ell_1, \ell_2, \ell_3)$, $\ell_* = \min(500, \ell_{min})$, $\zeta = \min(1/500, 1/\ell_{min})$, $A \sim (8\pi G\mu)^3$ and

$$L = \zeta\sqrt{\tfrac{1}{2}(\ell_1^2\ell_2^2+\ell_2^2\ell_3^2+\ell_3^2\ell_1^2)-\tfrac{1}{4}(\ell_1^4+\ell_2^4+\ell_3^4)}\,.$$





Regan & EPS, 2009; see also Ringeval et al 2009

- *Symmetry considerations suppress 3pt relative to 4pt*

Trispectrum constraints

Fergusson, Regan & EPS, arXiv:1012.6039

We have constrained a small selection of models via the trispectrum (normalised to g_{NL})

$$G_{NL}^{local} = 1.62 \pm 6.98 \times 10^5$$
$$G_{NL}^{const} = -2.64 \pm 7.20 \times 10^5$$
$$G_{NL}^{equi} = -3.02 \pm 7.27 \times 10^5$$
$$G\mu < 1.1 \times 10^{-6}$$

Prospects for Planck good - notably stringent cosmic string bound

Modal LSS estimator

- *Estimator for a theoretical bispectrum $B(k_1,k_2,k_3)$*

$$\mathcal{E} = \frac{(2\pi)^3}{N^2} \int \frac{d^3k_1}{(2\pi)^3}\frac{d^3k_2}{(2\pi)^3}\frac{d^3k_3}{(2\pi)^3}\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\frac{\delta^{obs}_{\mathbf{k}_1}\delta^{obs}_{\mathbf{k}_2}\delta^{obs}_{\mathbf{k}_3}B(k_1,k_2,k_3)}{P(k_1)P(k_2)P(k_3)}$$

- seems computationally intensive with $O(n_{max}^6)$ operations
- *$B(k_1,k_2,k_3)$* acquires extra NG from nonlinear gravity
- multiple N-body simulations/analysis for each model tested

- *Mode expansion for bispectrum (and trispectrum)*

$$\frac{\sqrt{k_1k_2k_3}B(k_1,k_2,k_3)}{\sqrt{P(k_1)P(k_2)P(k_3)}} = \sum \alpha^{\mathcal{Q}}_n Q(k_1,k_2,k_3)$$

- *Modal LSS estimator*

$$\mathcal{E} = \frac{1}{N^2}\sum \alpha^{\mathcal{Q}}_n \beta^{\mathcal{Q}}_n$$

FRS, arXiv: 1008.1730

Generic LSS Initial Conditions

- *N-body simulations with arbitrary bispectrum i.c.s*

$$\Phi^B(\mathbf{k}) = \int \frac{d^3\mathbf{k}'}{(2\pi)^3}\frac{d^3\mathbf{k}''}{(2\pi)^3}\frac{(2\pi)^3\delta(\mathbf{k}+\mathbf{k}'+\mathbf{k}'')B(k,k',k'')\Phi^G(\mathbf{k}')\Phi^G(\mathbf{k}'')}{P(k')P(k'')}\,,$$

$$= \sum_n \alpha_n \sqrt{\frac{P(k)}{k}}\, q_{\{r}(k)\int d^3\mathbf{x}\, e^{i\mathbf{k}.\mathbf{x}} M_s(\mathbf{x}) M_{t\}}(\mathbf{x}),$$

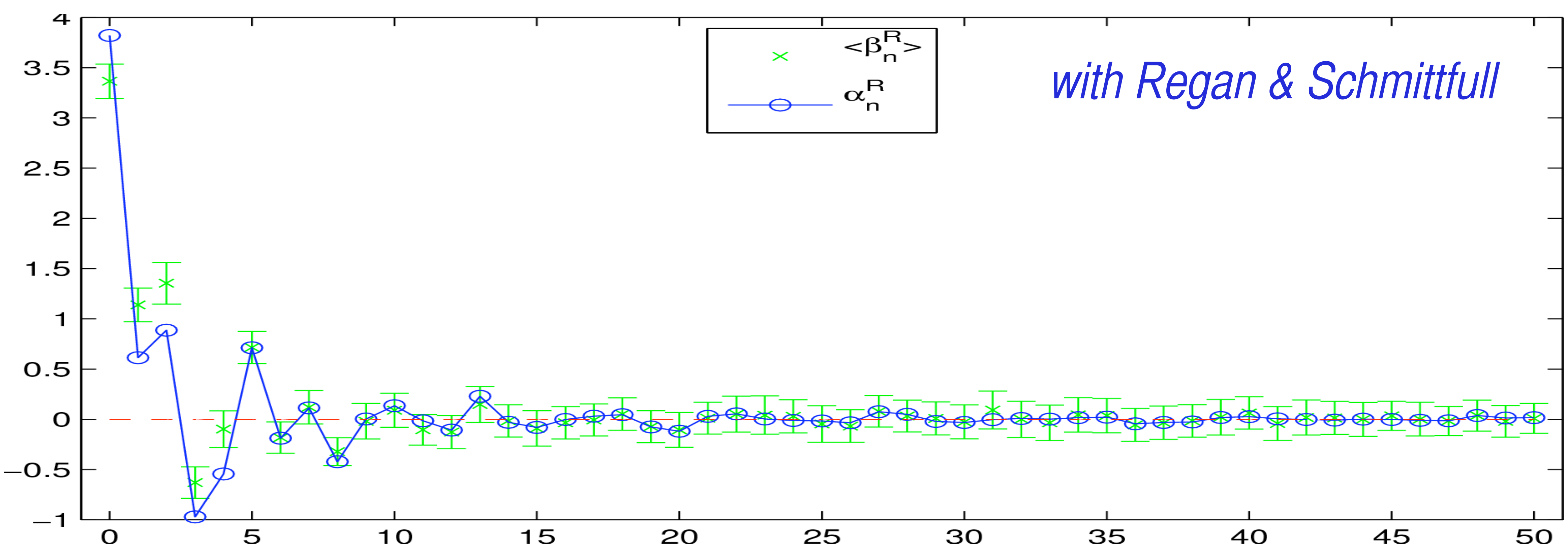
FRS, arXiv: 1008.1730

see also Verde et al. '10,'11

- *... and trispectra*

$$\Phi^T(\mathbf{k}) = \sum_n \bar{\alpha}_n^{\mathcal{Q}} q_r(k) \int d^3\mathbf{x}\, e^{i\mathbf{k}.\mathbf{x}} M_s(\mathbf{x}) M_t(\mathbf{x}) M_u(\mathbf{x})\,.$$

- *highly efficient working pipeline, e.g. local ...*



Intermediate Summary

- *Quantitative calculation of CMB bispectrum (in general case)*
- *Comp. tractable & robust   Applicable to cosmic strings, lensing etc*
- *New WMAP5 constraints on primordial models - total* F_{NL}

Model	$F_{\rm NL}$	$(f_{\rm NL})$
Constant	35.1 ± 27.4	(149.4 ± 116.8)
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Warm	24.2 ± 27.3	(94.7 ± 106.8)
Warm (Smoothed)	10.3 ± 27.2	(47.4 ± 125.4)

- *Constraints on wide range of feature models* $l^*>150$
- *New constraints on trispectrum (local, equil, and strings)*
- *Real discovery potential with Planck satellite at* $\Delta f_{NL} = 5$
- *Similar modal approach tractable LSS - truly generic i.c.s*